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PAYLOAD VOLUME OPTIMIZATION FOR SUBMARINE CONCEPT DESIGN

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THESIS

PAYLOAD VOLUME OPTIMIZATION FOR SUBMARINE CONCEPT DESIGN

by

Anthony Constable

September 2018

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Advances in payloads such as weapons and unmanned and autonomous vehicles need to be integrated into the Submarine Fleet to help maintain U.S. naval dominance. This thesis uses common submarine design equations to develop a model estimating a first-order balanced submarine design focused on hosting a range of payload concepts. The model uses an Integer Linear Program to maximize payload weight and return the optimal length and diameter for the submarine. The model is built and run in Excel Solver with the use of macros to facilitate multiple run conditions. Through the use of optimization, the impacts of payload capacity on basic submarine characteristics of length and diameter are assessed. Over 5,000 configurations of payload loadouts, ship lengths, and diameters are assessed in this thesis. The model outputs allow for trend analysis on the impacts of different payloads on ship length and diameter, provide optimal payload hosting locations on the submarine, and return optimal lengths and diameters for supporting specific payload loadouts. The modeling capability can be used as a decision aid in setting the overall submarine characteristics during the early stages of design without sacrificing payload capacity or flexibility.

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PAYLOAD VOLUME OPTIMIZATION FOR SUBMARINE CONCEPT DESIGN

Anthony Constable
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Submitted in partial fulfillment of the
requirements for the degree of

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from the

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ABSTRACT

Advances in payloads such as weapons and unmanned and autonomous vehicles need to be integrated into the Submarine Fleet to help maintain U.S. naval dominance. This thesis uses common submarine design equations to develop a model estimating a first-order balanced submarine design focused on hosting a range of payload concepts. The model uses an Integer Linear Program to maximize payload weight and return the optimal length and diameter for the submarine. The model is built and run in Excel Solver with the use of macros to facilitate multiple run conditions. Through the use of optimization, the impacts of payload capacity on basic submarine characteristics of length and diameter are assessed. Over 5,000 configurations of payload loadouts, ship lengths, and diameters are assessed in this thesis. The model outputs allow for trend analysis on the impacts of different payloads on ship length and diameter, provide optimal payload hosting locations on the submarine, and return optimal lengths and diameters for supporting specific payload loadouts. The modeling capability can be used as a decision aid in setting the overall submarine characteristics during the early stages of design without sacrificing payload capacity or flexibility.

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LIST OF ACRONYMS AND ABBREVIATIONS

ASDS	Advanced Seal Delivery Vehicle
AUV	autonomous underwater vehicle
DDS	Dry Deck Shelter
DoN	Department of the Navy
MAC	multiple all up round canister
NSC	near surface condition
PMB	parallel mid body
ROB	reserve buoyance factor
SSBs	ballistic missile submarines
SSGNs	guided missile submarines
SSNs	fast attack submarines
UUV	unmanned underwater vehicle
UUVMP	Unmanned Undersea Vehicle Master Plan
VPM	VIRGINIA payload module
WG	weight group

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EXECUTIVE SUMMARY

This thesis uses design equations provided by Burcher and Rydill (Burcher 1994) and Jackson (Jackson 1992) to develop a model estimating a first order balanced submarine design for providing the ship characteristics of length and diameter required to host a range of submarine deployed payloads. Submarine design begins with developing a balanced hydrostatic design, which is a design in which the overall weight of the vessel is equal to the overall buoyancy of the vessel in the submerged condition with buoyance defined as the weight of the water the submarine displaces. Overall, submarine characteristics (length and diameter) are driven during early phase design by mission requirements such as speed, depth, quieting, and desired payload capacity. Payload capacity can be a primary driver in required submarine volume and therefore it is often one of the first requirements to be relaxed. Payload fraction, a ratio of payload capacity to ship buoyancy, is a good metric for determining how much of the submarine is dedicated to the support of payloads. Most U.S. submarines have payload fractions of about 1%.

The thesis develops an Integer Linear Program to maximize payload weight, and thereby payload fraction, by determining the optimal length and diameter for the submarine. The model is built and run in Excel Solver with the use of macros to facilitate multiple run conditions. The optimization model developed as part of this thesis prescribes the best values for the number of payloads, location of payloads, submarine length, and submarine diameter that maximizes the payload fraction. To evaluate the model four payloads are selected: torpedoes, missiles, medium Unmanned Underwater Vehicles (UUVs), and large UUVs. In addition, four payload hosting locations are selected: two located internal to the pressure hull (internal rooms and internal large diameter tubes) and two located exterior to the pressure hull (external large diameter tubes and wet hangars). User-defined inputs are available to provide run conditions for desired payload loadouts. The model fidelity is hindered slightly by the constraint to use only open source data. However, the real value is in developing a process for using optimization in what has historically been a step-by-step iterative design process.

Over 5,000 configurations of payload loadouts, ship lengths, and diameters are assessed in this thesis. Initial trend analysis for individual payloads shows that as payload loadouts increase, the ship grows to accommodate and subsequently the payload fraction increases. However, as payload loadouts increase, the trends show diminishing returns as payload fractions approached 2%. Overall, the model returned expected values for length and diameter for payload loadouts that are consistent with existing U.S. submarine classes. The analysis of the optimization model shows that wet hangars provide the maximum efficiency with respect to payload fraction for three of the four payloads (torpedoes, medium UUVs and large UUVs). The optimal location for missiles is the internal large diameter tubes. The trends for individual payload stowage locations held true in more complex loadout cases with multiple payloads as well. Finally, the optimization model is able to consistently and rapidly provide results, which can produce useful data in aiding the decision process early in the submarine concept phase for assessing payloads capacity and flexibility.

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Burcher, Roy, and Louis Rydill. 1994. *Concepts in Submarine Design.* Cambridge, MA: Cambridge University Press.

Jackson, Harry A. 1992. "Fundamentals of Submarine Concept Design." *The Society of Naval Architects and Marine Engineers Transactions* 100: 419–448.

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I. INTRODUCTION

In 2015, Secretary of the Navy Ray Mabus stressed the importance of the Navy mission in an update to the SEAPOWER 21, which sets a vision for the Navy in the 21st century and identifies the importance of maintaining sea control, stating, "The essential elements of sea control are surface warfare, undersea warfare, strike warfare, mine warfare, air and missile defense, maritime domain awareness, and intelligence, surveillance and reconnaissance" (Department of the Navy [DoN] 2015, 22). Today's U.S. Navy submarines play a critical role in maintaining sea control and due to their stealth attributes will play an ever-increasing role in the 21st century. The advances in weapons and unmanned and autonomous vehicles "[allow] naval forces to establish local maritime superiority while denying an adversary that same ability" (DoN 2015, 22). Integrating these capabilities into the U.S. submarine fleet is a key enabler in meeting the SEAPOWER 21 goals.

Overall, submarine characteristics (length and diameter) are driven during early phase design by mission requirements such as speed, depth, quieting, and desired payload capacity. Throughout the early stages of design, tradeoffs between mission requirements must be made. Payload capacity can be a primary driver in required submarine volume and therefore is often one of the first requirements to be relaxed. The result has been submarine designs with payload hosting capability that is only sufficient to meet the initial mission requirements with minimal flexibility for growth or reconfiguration.

To maintain alignment with the vision of SEAPOWER 21, future missions for submarines will likely put an increased emphasis on payload capacity (hosting improved weapons, manned vehicles, and unmanned vehicles). This thesis provides methods to investigate trade space in payload capacity early in the submarine design phase. Through the use of optimization, the impacts of payload capacity on basic submarine characteristics of length and diameter are assessed. The purpose is to provide modeling capability that can be used as a decision aide in setting the overall submarine characteristics during the early stages of design without sacrificing payload capacity or flexibility.

A. CURRENT UNITED STATES SUBMARINES FLEET

U.S. submarines fall into two categories: fast attack submarines (SSNs) and ballistic missile submarines (SSBNs). Each type is designed and operated based on traditional missions that have not changed much since the Cold War. Fast attack submarines generally serve the purpose of undersea warfare and are smaller and nimbler. Ballistic missile submarines serve the purpose of strategic deterrence and are larger to accommodate nuclear missiles.

1. Nuclear-Powered Attack Submarines

Nuclear powered attack submarines, referred to as SSNs, make up a majority of the U.S. Navy submarine fleet. Currently there are three active classes of SSNs in the U.S. fleet: the Los Angeles class, the Seawolf class, and the Virginia class (shown in Figure 1). SSNs are designed to support multiple missions, which include: anti-ship and anti-submarine warfare, Tomahawk launch, intelligence gathering and recognizance, special operating forces, and mine warfare. The major emphasis in the design of the modern-day SSN was quiet operations and speed with the goal of maintaining undersea superiority over foreign adversaries.

Virginia class submarine Block 1 at Sea, one of three active classes of nuclear powered attack submarines in the U.S. Navy. It is capable of carrying torpedoes and Tomahawk missiles.

Figure 1. Virginia Class Attack Submarine. Source: Navy (2017a).

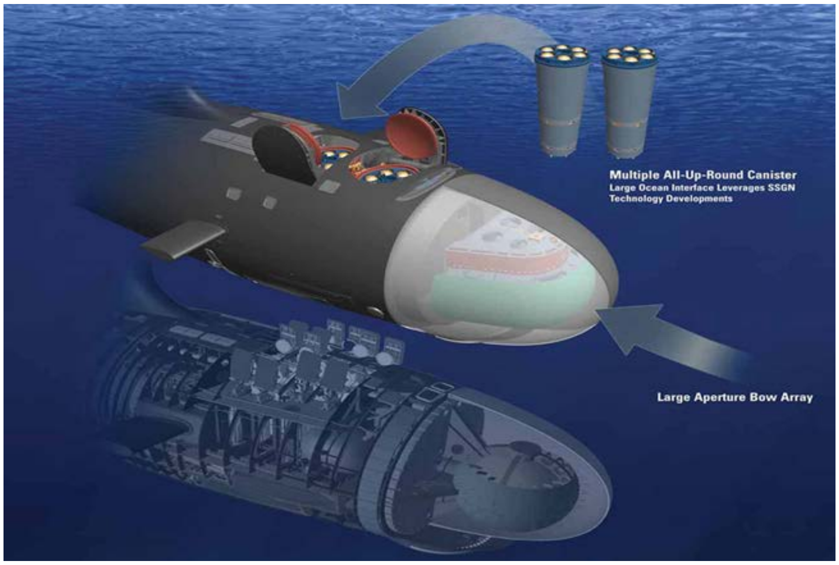
The Los Angeles class was built and delivered between 1972 and 1996 (Sharpe 1997). It has a length of 362 feet, a diameter of 33 feet, and a displacement of 6,082 tons (Sharpe 1997). These boats were the primary defense against the Soviet Navy during the Cold War. The Los Angeles class is equipped with four torpedo tubes capable of launching Mk 48 heavyweight torpedoes as well as Tomahawk land attack cruise missiles. Later upgrades to the class incorporated vertical launch tubes for carrying Tomahawk external to the pressure hull. Each Tomahawk was stored in its own missile tube. The Los Angeles class is also capable of hosting vehicles to support Special Operational Forces (Navy 2003). These include the Dry Deck Shelter (DDS) and the Advanced Seal Delivery Vehicle (ASDS), shown in Figure 2, which can be attached to the top of the submarine external to the pressure hull.

Los Angeles class submarine at sea equipped with the Advanced Seal Delivery System for Special Forces.

Figure 2. ASDS on a Los Angeles Class Submarine. Source: Navy (2003).

The Seawolf class was built and delivered between 1989 and 2001 (Sharpe 1997). It has a length of 353 feet, a diameter of 40 feet, and a displacement of 7,460 tons (Sharpe 1997). These boats were designed with improved quieting and advanced technology with the goal of regaining the undersea warfare advantage against the Soviet Navy. The larger diameter of 40 feet compared to the 33 foot Los Angeles provides additional capacity for weapons. The Seawolf class is equipped with eight torpedo tubes (versus four for Los Angeles) and vertical launch tubes for carrying Tomahawks. The Seawolf design was discontinued after three hulls due to cost overruns.

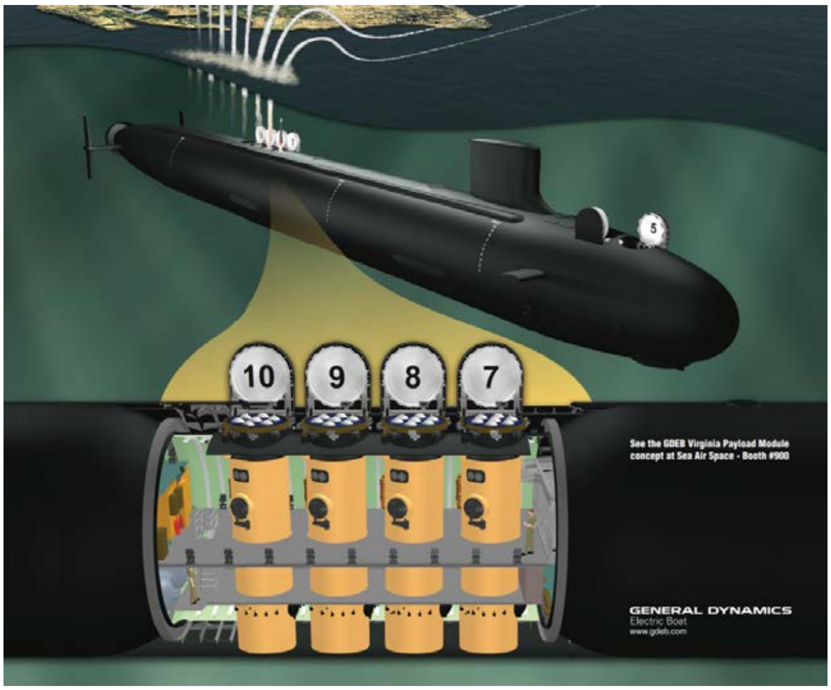
The Virginia class started production in 1998 and is still in production today (Sharpe 1997). It represents the most advanced submarine in the U.S. Navy and was designed to leverage the Seawolf capabilities in an affordable design. It has a length of 377 feet, a diameter of 34 feet, and a displacement of 7,700 tons (Sharpe 1997). The first two Blocks of Virginia have similar armaments as Los Angeles (four torpedo tubes and vertical launch tubes of Tomahawks). Starting with the Block three, the bow is redesigned to change the individual vertical launch tubes into two large diameter tubes (as shown in Figure 3). The large diameter tubes are more capable than the vertical launch tubes due to the added volume they provided. Each large diameter tube can host up to seven Tomahawks which are loaded into the tubes in interfacing structure called Multiple All Up Round Canisters (MACs) which can be described as a bullet magazine for missiles (Strategic Systems Programs n.d.). The Virginia class is also capable of hosting the DDS and the ASDS in a similar manner to the Los Angeles.



Virginia class submarine Block 1–2 with vertical launch tubes vs. the Block 3 upgrade with the two large diameter tubes.

Figure 3. Tomahawk Launcher (Large Diameter Tube vs. Vertical Launch Tubes). Source: Foxtrot Alpha (2015).

The fifth Block of Virginia class submarines is being designed with an additional 90 feet of ship length referred to as the Virginia Payload Module (VPM) (GDEB n.d.). The VPM (shown in Figure 4) will include four additional large diameter tubes internal to the pressure hull designed for hosting up to 28 additional Tomahawk missiles. These tubes also provide potential options for hosting other payloads.



Virginia class Submarine Block 5 conceptual drawing showing the addition of four large diameter vertical tubes designed for Tomahawk launch.

Figure 4. Virginia Payload Module. Source: GDEB (n.d.).

2. Nuclear-Powered Ballistic Missile Submarines

Nuclear powered ballistic missile submarines, referred to as SSBNs, are considered one of the primary strategic assets to the United States nuclear deterrent. Currently, the Ohio class (Figure 5) encompasses the entire SSBN fleet for the U.S. Navy. The SSBNs are designed for the sole mission of carrying the Trident ballistic missile. Due to the quiet operation of the Ohio class, the submarine can remain virtually undetected which is why it is viewed as the primary arm of the nuclear deterrent.

Ohio class submarine at sea. The Ohio class is the current ballistic missile submarine used for strategic deterrence in the U.S. Navy.

Figure 5. Ohio Class Ballistic Missile Submarine. Source: Navy (2017b).

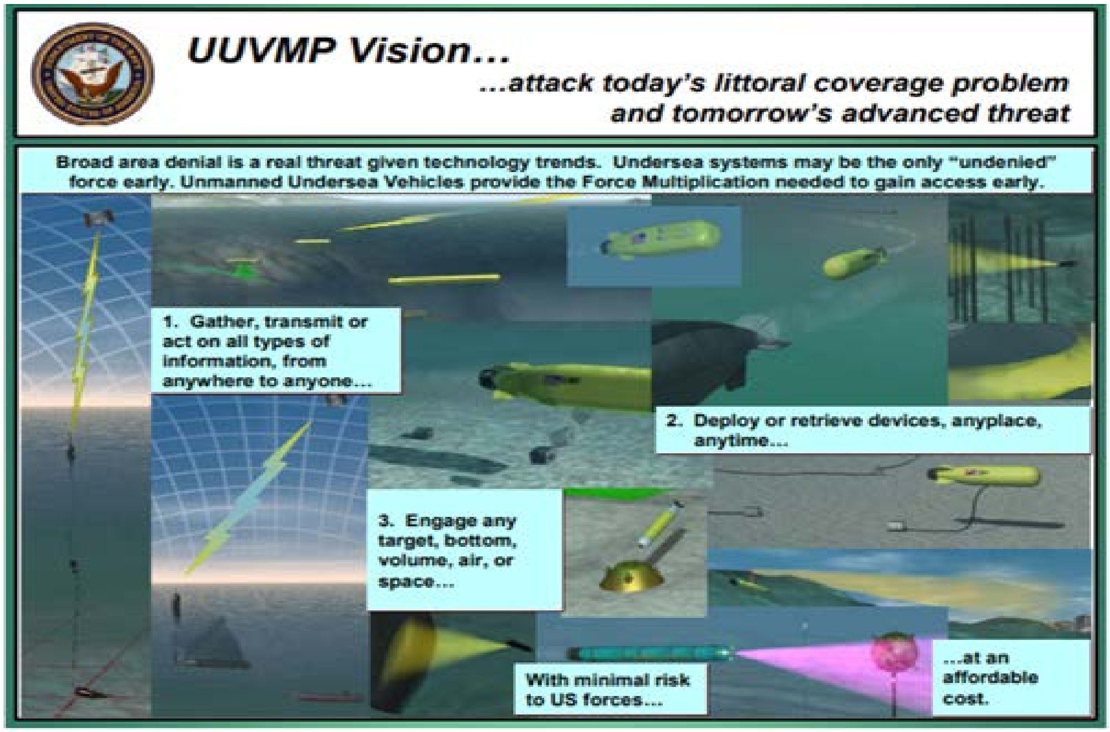
The Ohio class was built and delivered between 1979 and 1997 (Sharpe 1997). It has a length of 560 feet, a diameter of 42 feet, and displacement of 16,600 tons (Sharpe 1997). These boats served as the primary deterrent to the nuclear armed Soviet Navy during the Cold War and continue in the function today. The Ohio class is equipped with four torpedo tubes and twenty-four vertical missile tubes for hosting Trident nuclear missiles. The entire design of the Ohio class was driven by the Trident missile.

In 2002, the Navy began to convert four of the Ohio class SSBNs to support non-strategic deterrent missions (Strategic System Program n.d.). The missile tubes where altered to support launching Tomahawk missiles using MACs. In addition, extensive modifications were made to allow the Ohio to host Special Operational Forces vehicles (DDS and the ASDS). The four converted SSBNs were given the designation of SSGNs where the “G” refers to guided missiles in lieu of ballistic missiles.

B. FUTURE SUBMARINE PAYLOADS

One of the primary purposes of a submarine is to carry payloads to support undersea missions. These payloads can be weapons, manned vehicles, and unmanned vehicles. Traditionally, submarines are designed around weapon payloads. Attack submarines are designed to carry torpedoes and Tomahawk missiles while ballistic missile submarines are designed to carry nuclear missiles. Hosting of manned vehicles has been limited to piggyback designs like the DDS and ASDS. Piggyback designs serve the intent, but are not efficient with respect to ship transit speeds or access and maintenance to the vehicles while on mission. Hosting of unmanned vehicles on submarines tends to be limited to existing ship interfaces such as the torpedo tubes.

The future vision for undersea warfare includes extensive use of unmanned underwater vehicles (UUVs) and autonomous underwater vehicles (AUVs) launched from submarine platforms. In 2004, the U.S. Navy issued the *Navy Unmanned Undersea Vehicle Master Plan (UUVMP)*, which describes the importance of UUVs in maintaining maritime superiority long into the future (DoN 2004). Figure 6 shows the vision of using UUVs for extending the Navy’s reach in the future.

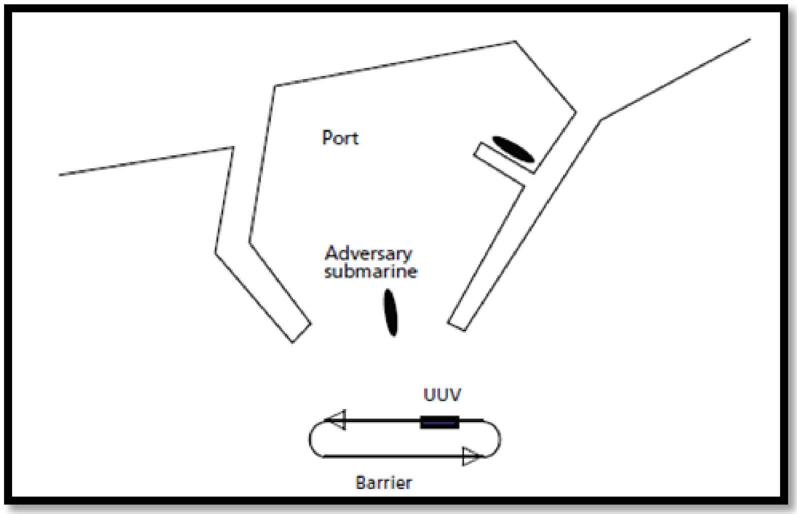


Vision of the Undersea Vehicle Master Plan (UUVMP) that describes the importance of UUVs in maintaining maritime superiority long into the future.

Figure 6. UUVMP Vision (UUVMP). Source: DoN (2004).

The UUVMP identified the following nine capabilities for achieving the vision for the future use of UUVs: 1. Intelligence, Surveillance, and Reconnaissance 2. Mine Countermeasures 3. Anti-Submarine Warfare 4. Inspection / Identification 5. Oceanography 6. Communication / Navigation Network Node 7. Payload Delivery 8. Information Operations 9. Time Critical Strike (DoN 2004). UUVs and AUVs offer unique and covert ways to accomplish these missions. UUVs and AUVs are smaller and less detectable than full-scale submarines. In addition, UUVs and AUVs can be considered more expendable than manned submarines and have the added advantage of reducing the risk to the host platform and the sailors.

In 2009, the RAND Corporation conducted a survey of missions for unmanned undersea vehicles (Button et al. 2009). The purpose of the study was to assess "which missions for UUVs appear the most promising to pursue in terms of military need, risk, alternatives, and cost" (Button et al. 2009, 8). One example listed by RAND was the use of UUVs for harbor monitoring. In Figure 7, a UUV or swarm of UUVs is used to hold an enemy submarine at risk as it attempts to leave its homeport. This is an excellent example of a UUV conducting a mission that would be high risk if conducted by a manned submarine.

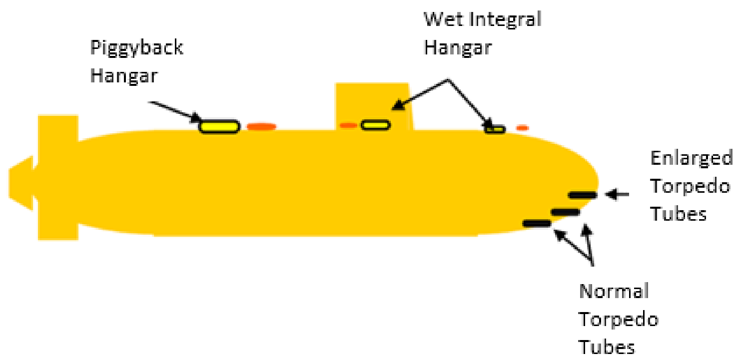


Potential mission for UUVs describing the use of UUVs to hold an enemy submarine at risk while the submarine attempts to exit it homeport.

Figure 7. UUV Hold-at-Risk Anti-submarine Warfare. Source: Button et al. (2009).

A key to utilizing UUVs and AUVs in contested and forward deployed waters is transiting them to theatre and supplying maintenance capabilities. The RAND study identified that UUVs have a typical operating time on the order of days (Button et al. 2009). Transit distances for UUVs are limited due to power density (most UUVs are power by lithium ion batteries or some variation). The RAND study also identified that UUV reliability has not been well demonstrated with limited real world military operations (Button et al. 2009). Deployed UUVs and AUVs must display high levels of reliability to ensure mission failure does not occur. Due to the limited operation time and the untested reliability of UUVs and AUVs, it is prudent to support UUV and AUV missions with forward deployed vessels to serve as "motherships." Nuclear submarines can serve as good host platforms because they can operate forward undetected where they can deploy, retrieve, and service UUVs and AUVs. However, designing a submarine to support the complex operations of deploying and retrieving UUVs and AUVs is not a trivial task.

In 2008, BMT Group Ltd presented a paper on design considerations for UUV launch and retrieval (Hardy and Barlow 2008). The study was focused on modifying existing submarines for UUV operations. Hardy and Barlow identified five methods for UUV launch and recovery as shown in Figure 8. Method one involved utilizing standard torpedo tubes, method two utilizes larger diameter torpedo tubes, method three involves dry piggyback hangars (e.g., the DDS), method four involves wet piggyback hangars, and method five includes more specialized wet hangars located strategically around the submarine (Figure 9).



Potential hosting locations on submarines for the purpose of deploying and retrieving UUVs. Locations include normal and enlarged torpedo tubes, large piggyback hangars, and wet integral hangars.

Figure 8. Options for UUV Deployment. Source: Hardy and Barlow (2008).

Conceptual design of a wet hangar interface for UUV on a Submarine. Each hangar hosts a single torpedo like payload and the hangar is faired into the ship structure.

Figure 9. Conceptual Specialized Wet UUV Hangars. Source: Hardy and Barlow (2008).

C. OBJECTIVE OF STUDY

Future use of UUVs and AUVs in concert with U.S. Navy submarines is a multifaceted problem as this creates requirements not only on the UUV or AUV but also on the host submarines, and logistic systems that support them. Host platforms must provide storage for UUVs and AUVs as well as sufficient space for spares, alternate payloads, and work space to perform servicing. UUVs and AUVs must be designed to consider the workspace, tools, and skills available on forward deployed vessels. These constraints often drive systems to require high reliability at high cost.

Historic submarine design practice has been built on optimizing a submarine design around a defined specific payload. Once the payload is set, the trade space in the submarine design is driven by factors such as speed and quieting instead of payload. This does not allow for designs to be optimized for future needs of payload hosting, such as adding UUVs or AUVs. The first stage of submarine design is to create a balanced submarine design that guarantees there is sufficient volume and ballast for the submarine. This critical step sizes the pressure hull of the submarine and ensures its capability to submerge and surface. This thesis investigates the weight and volume design requirements for hosting and servicing UUVs and AUVs in the forward theatre.

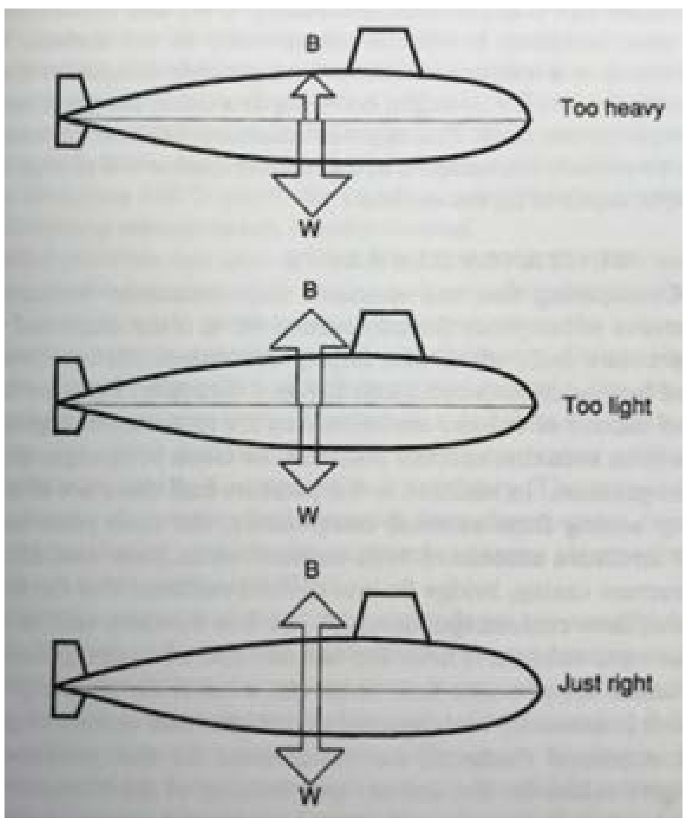
The U.S. Navy's future plans include extensive use of UUVs and UAVs. However, payload volume requirements for UUVs and UAVs on future submarine designs are ill-defined and result in inefficient submarine designs for payload hosting. This research provides benefits in helping guide payload volume requirement definition for future platforms. First, this thesis uses optimization to show trends in payload capacity over a range of submarine lengths and diameters for selected payload types and hosting interfaces. These trends may be used as a decision aide in early stage design in selecting a range of submarine characteristics. Secondly, the optimization is used to determine the optimal length and diameter for a user-defined payload loadout.

Chapter II reviews submarine design practice, definition of payload fraction and descriptions of existing and potential submarine payloads. Chapter III is a description of the optimization model. Chapter IV discusses the analysis methods and results. Chapter V provides overall conclusions and recommendations for future research.

II. BACKGROUND

A. COMMON SUBMARINE DESIGN PRACTICE

Roy Burcher and Louis Rydill published *Concepts in Submarine Design* in 1994, which provides a high-level process of submarine concept design. Submarine design begins with developing a balanced hydrostatic design, which is a design where the overall weight of the vessel is equal to the overall buoyancy of the vessel in the submerged condition. This condition is referred to as being neutrally buoyant as shown in Figure 10. Submarine design has inherently been an iterative process. It typically takes multiple iterations for the design to reach a point where the total weight is matched by the available buoyancy.



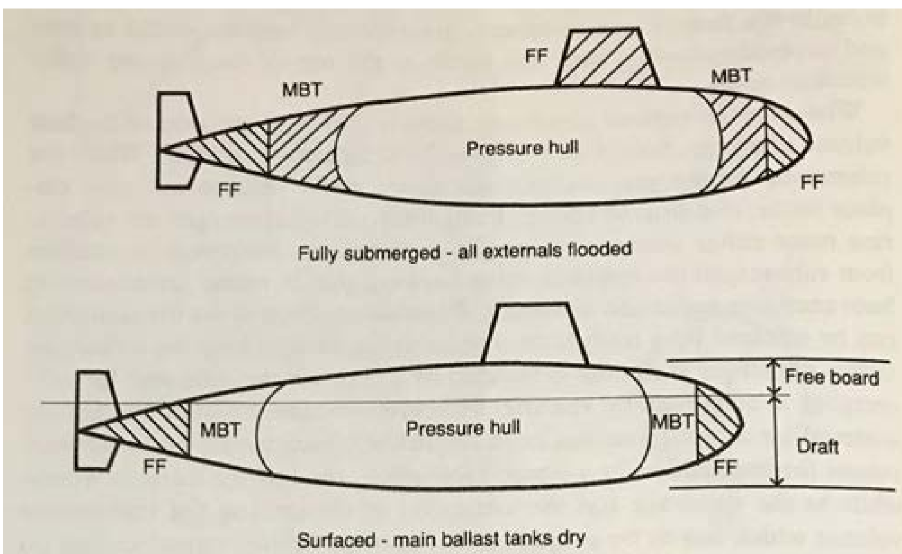
Design requirement to achieve neutral buoyancy for submarines. If weight is more than buoyance, the submarine sinks, if weight is less than buoyancy the submarine rises.

Figure 10. Neutral Buoyancy. Source: Burcher and Rydill (1994).

Submarine buoyance is determined by the weight of the water the submarine displaces. Burcher and Rydill (1994) discusses how a submarine's total or envelope volume (V_{form}) is made up a three types of volumes: pressure hull volume (V_{PH}), main ballast tanks volume (V_{MBT}), and free flood volume (V_{FF}), as shown in Equation (1.1).

$$V_{form} = V_{PH} + V_{MBT} + V_{FF} \tag{1.1}$$

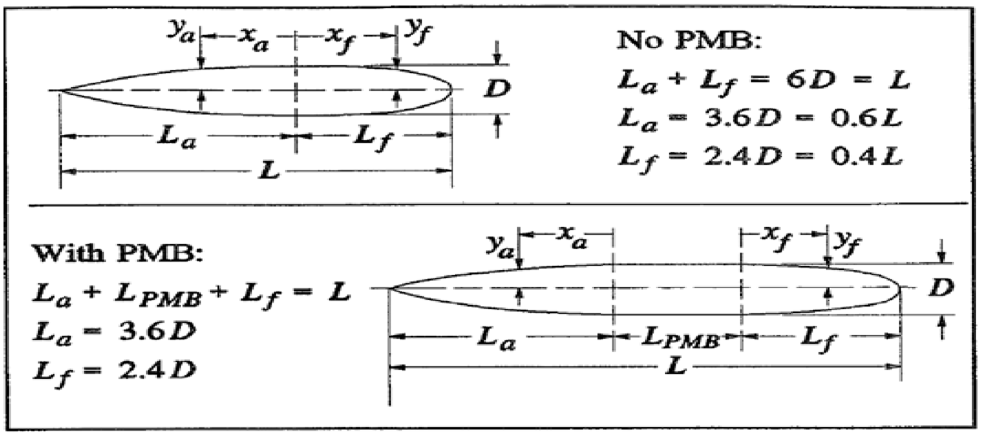
The pressure hull volume is the dry side of the submarine often referred to as the people tank. The main ballast tank volume is the tanks that are filled with seawater to submerge the submarine. The free flood volume is the area outside the pressure hull volume that cannot easily be sealed from the sea, such as the areas around the rudders. To determine the amount of water that is displaced by a submerged submarine, the volume of the submarine taken up by seawater is subtracted from the volume of the hull. The relationship between the pressure hull, main ballast tanks, and free flood are shown in Figure 11.



A submarine's total or envelope volume (V_{form}) is made up a three types of volumes: pressure hull volume (V_{PH}), main ballast tanks volume (V_{MBT}), and free flood volume (V_{FF}). When the submarine is submerged the ballast tanks are filled and when the submarine is surfaced, the ballast tanks are emptied.

Figure 11. Submarine Volumes: Surfaced and Submerged. Source: Burcher and Rydill (1994).

The total volume of the submarine of V_{form}, is a function of the length (L) and diameter (D) of the submarine. CAPT Harry Jackson published a paper in 1992 entitled “Fundamental of Submarine Concept Design” in which he discusses submarine concept design. Optimal design for a submerged body of revolution for flow through the water would dictate a length to diameter ratio of approximately six (Jackson 1992). This works from a ship resistance perspective but is not sufficient to support the internal volumetric requirements for submarines; therefore, a parallel mid body (PMB) is often added which has a constant diameter to increase arrangeable volume. Length to diameter ratios larger than 15 are not common in current submarine designs. Jackson (1992) developed a typical submarine shape with a ratio where the forward 40% of the length (L_f) is parabolic in nature while the aft 60% of the length (L_a) is elliptical. Length overall is the sum of L_f, L_a, and any added PMB identified as L_{PMB}. Figure 12 shows this relationship. Jackson (1992) offers Equations (1.2) and (1.3) to estimate the radius of the hull (y_f and y_a respectively) along the ship length at distances from the PMB (x_a and x_f). Form factors for the shape of the forward and aft end dictating the fullness of the parabolic and elliptical shapes and are identified as n_f and n_a. For consistency with Jackson (1992), n_f is set to 3 and n_a is set to 2.75 for this thesis.



The total volume of the submarine is a function of the length (L) and diameter (d) of the submarine. Optimal design for a submerged body of revolution would dictate a length to diameter ratio of approximately six. Parallel mid body (PMB) is often added to increase arrangeable volume. CAPT Harry Jackson developed a typical submarine shape with a ratio where the forward 40% of the length (L_f) is parabolic in nature while the aft 60% of the length (L_a) is elliptical. Length overall is the sum of L_f, L_a, and any add PMB identified as L_{PMB}. The radius of the hull (y_f and y_a respectively) can be calculated from points from the PMB (x_a and x_f).

Figure 12. Geometry of Submarine. Source: Jackson (1992).

$$y_f = \frac{D}{2}\left[1-\left(\frac{x_f}{L_f}\right)^{n_f}\right]^{\frac{1}{n_f}} \tag{1.2}$$

$$y_a = \frac{D}{2}\left[1-\left(\frac{x_a}{L_a}\right)^{n_a}\right] \tag{1.3}$$

The total volume of the submarine or V_{form} can be calculated assuming the outline of the hull as defined by y_f and y_a is a body of revolution around the main axis. To estimate the V_{form}, an adaptation of the Trapezoid Rule as presented in *Principles of Naval Architecture* (Lewis 1988) can be used as shown in Equation (1.4) by breaking up the submarine length into one-foot increments and summing over the length of the submarine. By combining equation (1.4) with Equations (1.2) and (1.3), the submarine's volume can be estimated with Equation (1.5).

$$V_{form} = \sum_{f=0}^{Lf} \pi y_f^2 + \sum_{p=0}^{Lpmb} \pi y_{pmb}^2 + \sum_{a=0}^{La} \pi y_a^2 \tag{1.4}$$

$$V_{form} = \sum_{x_f=0}^{3.6D} \pi \left(\frac{D}{2} \times \left[1 - \left(\frac{x_f}{3.6 \times D} \right)^{n_f} \right]^{\frac{1}{n_f}} \right)^2 + \sum_{0}^{L-6D} \pi \left(\frac{D}{2} \right)^2 + \sum_{x_a=0}^{2.4D} \pi \left(\frac{D}{2} \times \left[1 - \left(\frac{x_a}{2.4 \times D} \right)^{n_a} \right] \right)^2 \quad (1.5)$$

Burcher and Rydill (1994) offers the following relationships between V_{form}, V_{PH}, V_{MBT}, and V_{FF}. Equation (1.6) shows the V_{MBT} is based on V_{PH} where the reserve buoyancy fraction (*ROB*) is determined by the designer and a utility factor (typically 0.98) that is included to account for the internal structure of the main ballast tanks. A *ROB* of 12.5% is fairly typical for submarine designs (Burcher and Rydill 1994). Equation (1.7) shows the V_{form} as a function of V_{PH} and V_{MBT}. In this case, Burcher and Rydill uses a factor of 15% to account for the V_{FF}. Equations (1.6) and (1.7) are combined to solve for V_{PH} as a function of V_{form} in Equation (1.8). The submarine buoyancy is equal to V_{PH} divided by 35 ft^3/ton.

$$V_{MBT} = (V_{PH} \times ROB) / UtilityFactor \quad (1.6)$$

$$V_{form} = (V_{PH} + V_{MBT}) \times 1.15 \quad (1.7)$$

$$V_{PH} = \frac{V_{form}}{1.15 \times (1 + ROB / 0.98)} \quad (1.8)$$

For a balanced design, the sum of the weight of the submarine would equal the submarine buoyancy (V_{PH} / [35 ft^3/ton]). The submarine total weight is broken down into seven weight groups, which account for the constant vessel weight (Jackson 1992). These weight groups (shown in Table 1) are: Hull Structure, Machinery, Electrical Systems, Command and Control, Auxiliary Systems, Outfitting, and Payloads. The sum of these weight groups represent the constant weight of the submarine and is referred to as Condition “A-1.” These weight groups are mainly driven by mission requirements. Lead ballast is added to Condition "A-1" and the resulting weight is referred to as Condition “A.” Variable loads which include the crew, ships stores, torpedoes, and other payloads are added to Condition “A” to represent the Near Surface Condition (*NSC*) which is when the submarine is on the surface. Variable Ballast (water added to submerge the vessel) in the Main Ballast Tanks (MBTs) is added to *NSC* to achieve the Submerged Displacement (SUBD) condition. Finally, free-flood areas (places outside the pressure hull that cannot be sealed off from the sea) is added to the SUBD to achieve the Envelope Displacement (ENVD) condition. Table 1 shows the relationships among these conditions for a balanced design. For a balanced design, *NSC* would equal the displacement of the pressure hull V_{PH} / [35 ft^3/ton].

Table 1. Submarine Weight Balance. Adapted from Jackson (1992).

Weight Balance Title	**Symbol**	**Description**
Weight Group 1	WG_1	Hull Structure (pressure hull, non-pressure hull, framing, decks, hatches, etc.)
Weight Group 2	WG_2	Mechanical (propulsion system, nuclear reactor, etc.)
Weight Group 3	WG_3	Electrical (generators, power conversion equipment, panels, lighting, etc.)
Weight Group 4	WG_4	Command/Control (navigation, radio room, sonar equipment, etc.)
Weight Group 5	WG_5	Auxiliary (heating and air conditioning, refrigeration, plumbing, hydraulics, water systems, etc.)
Weight Group 6	WG_6	Outfitting (ladders, deck plating, galley, living spaces, etc.)
Weight Group 7	WG_7	Payloads (structure and systems to support payload, the payload themselves are consider variable loads)
Condition A-1	*A-1*	Sum of $WG_{(1\text{-}7)}$ – constant weight of the submarine
Lead	*Ld*	Lead Ballast added for hydrostatic purposes
Condition A	*A*	Sum of *A-1* + *Ld*
Variable Load	*VL*	Load which can change regularly (crew, stores, potable water, torpedoes, and other payloads
Near Surface Condition	*NSC*	Sum of *VL* + *A* (weight of the vessel while surfaced)
Main Ballast Tanks	*MBT*	Tanks filled with water to submerge the vessel
Submerged Displacement	*SUBD*	Sum of *NSC* + *MBT*
Free Flood	*FF*	Spaces within the hull lines not sealed off from the ocean
Envelope Displacement	*ENVD*	Sum of *SUBD* + *FF*

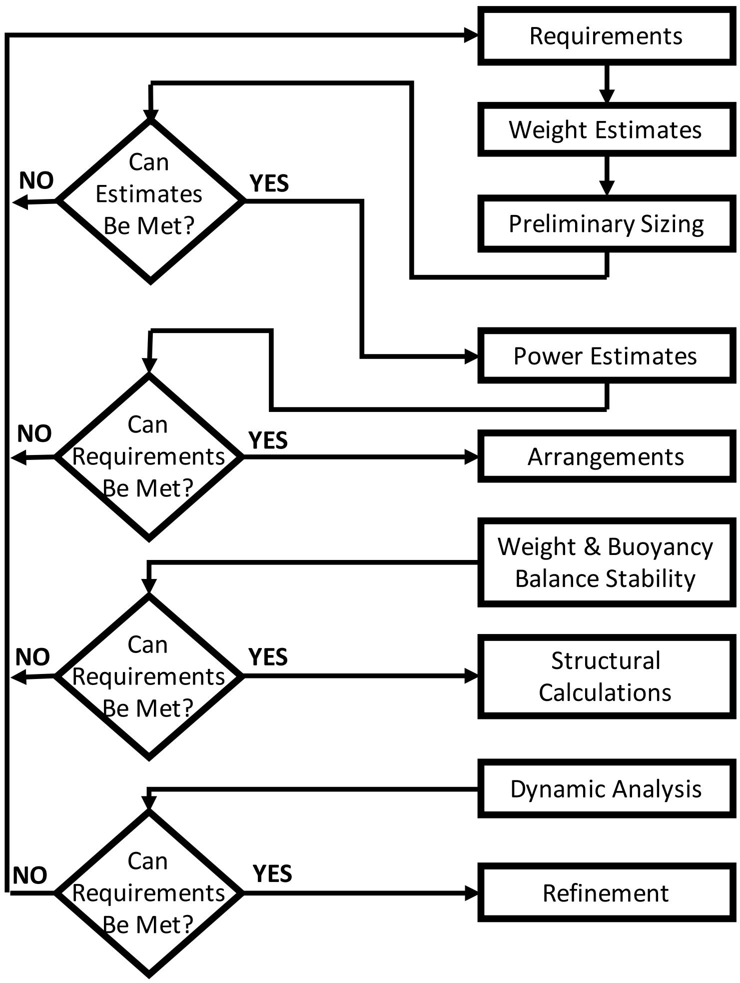
Submarine weight categories for establishing a ballasted design. Weight Groups 1 through 7 account for the constant design weight of the submarine. Variable loads consist of weights that change over time such as food, people, weapons, ballast tanks and free flood weight account for water taken onto the submarine external to the pressure hull.

B. SUBMARINE CONCEPT DESIGN

During concept design, there are different proposed methods to approach weight estimation. Jackson (1992) suggests starting with requirements for a weight-based estimate. Weight groups are estimated based on parametric data from similar submarines. Jackson suggests using the following relationship (Equation (1.9)) for calculating *NSC* from the Weight Groups (WG_i), the Lead (*Ld*), and the Variable Ballast (*VL*) described in Table 1.

$$NSC = \frac{[1+\%Ld]\times\sum_{i=2}^{7} WG_i}{1-\%VL-[\%WG_1\times[1+\%Ld]]} \tag{1.9}$$

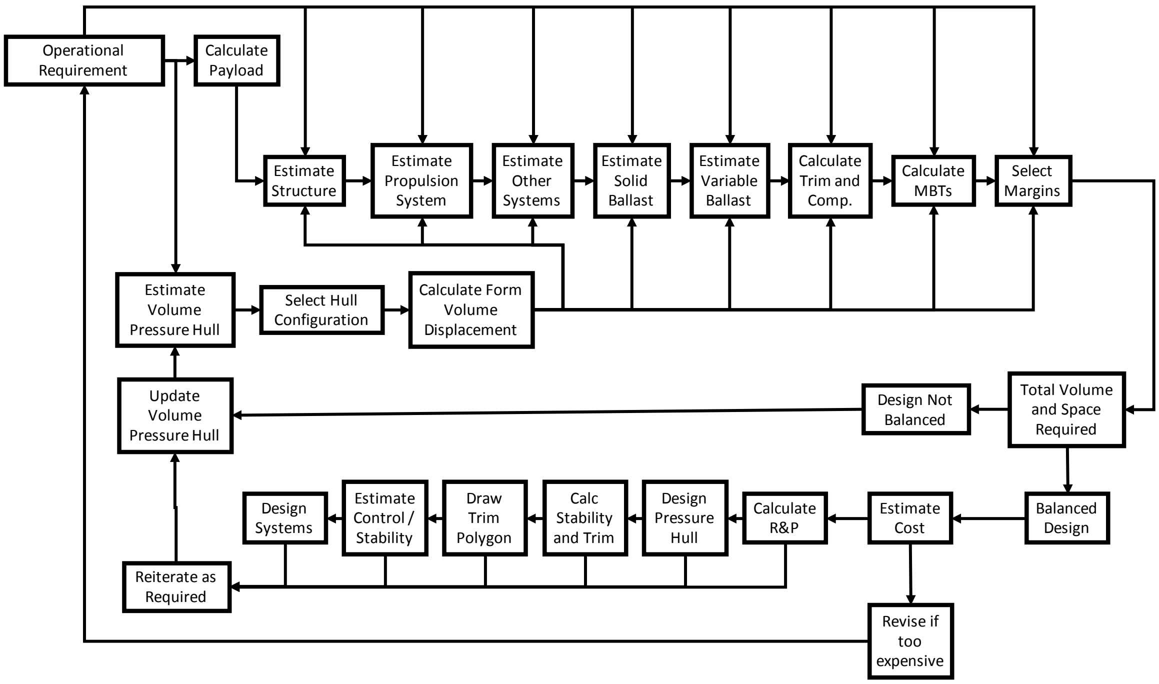
Once weight groups are estimated and the *NSC* calculated, the submarine length and diameter are determined to provide the required volume as shown in Figure 13. The volume is then checked against ship powering requirements and pressure hull arrangeable volume, while working towards a final design. If at any point during the design steps shown in Figure 13 that a requirement is not met, the process is restarted.



Submarine design process suggest by CAPT Harry Jackson utilizing a weight estimates for initial concept feasibility.

Figure 13. Submarine Feasibility Study Flow Chart. Adapted from Jackson (1992).

Another way to start concept design is with volume estimates instead of the weight-based approach as shown in Figure 14. In this method, suggested by Burcher and Rydill (1994), ship length and diameter are estimated using initial requirements for payloads. This is done parametrically. Then volumetric values for each of the weight groups in Table 1 are estimated starting with WG_2 and WG_3 for power which is based on speed requirements. These volumes are then placed inside the area allowed by the given ship length and diameter to check for a balanced design. The design follows a standard design spiral until a balanced design is obtained.



Submarine design process suggested by Burcher utilizing volume estimates for initial concept feasibility.

Figure 14. Submarine Concept Design Process. Adapted from Burcher and Rydill (1992).

Both methods proposed by Jackson and Burcher and Rydill are valid. However, both leverage the assumption of clear requirement definition of the payload upfront. This results in a design that is optimal given a particular payload (e.g., a missile system). Once the payload is accommodated, the other ship requirements (such as speed or depth) drive the ship design. Historically, this has resulted in limited flexibility in payload hosting and a relatively low ratio of payload weight to ship displacement often referred to as payload fraction.

C. PAYLOAD FRACTION

Payload fraction is defined as the ratio of total weight of payloads to surface displacement. Payloads include weapons such as missiles and torpedoes as well as deployable payloads such as unmanned underwater vehicles, autonomous unmanned vehicles, or special operational forces vehicles. The use of payload fraction is a good metric as it can help measure the mission capacity/flexibility of a submarine. Once a submarine expels all of its payloads, it typically requires a port call for replenishment. Maximizing payload load out for a platform allows maximum flexibility and capacity for missions. Traditionally, submarine payload capacity has been increased by increasing submarine displacement. Submarine displacement is a good proxy for cost, so as a submarine gets bigger, its cost increases accordingly. When designing a weapon system such as a submarine, maximizing payload while minimizing displacement should be the goal. The use of payload fraction as a metric can help assess the “bang for the buck” of a submarine design.

1. Fast Attack Submarine Payload Fraction

Fast attack submarines are built for anti-ship and anti-submarine warfare. Payload capacity competes with other design parameters such as speed and stealth. The Virginia Block 3 with 24 torpedoes at ~3400 lbs each (Seaforces n.d.) and 12 Tomahawks at ~3300 lbs each (Navy 2018) has a total payload weight of ~55 tons. Dividing this by the Virginia *NSC* of 7700 tons (Sharpe 1997) results in a payload fraction of ~0.7%. This is similar to all current U.S. attack boats with payload fraction around 1%. Many of the payloads are limited to standard ship interfaces such as torpedo tubes and vertical missile tubes. The addition of more modern payloads such as UUVs or AUVs requires the payload to be designed around these traditional ship interfaces. This limits the potential of alternate payloads as well as configurability and mission flexibility.

2. Ballistic Missile Submarine Payload Fraction

Ballistic missile submarines are built around a specific payload, ballistic missiles. A missile compartment is designed and integrated with the rest of the submarine. The Ohio SSBN has 24 Trident missiles at 130,000 lbs (Missile Threat 2016) for a payload weight of ~1400 tons. Dividing this by the Ohio *NSC* of 16,600 tons (Sharpe 1997) results in a payload fraction of ~8%. This higher payload fraction is a function of the heavy weight of the missile compared to other payloads. When four ballistic missile submarines were converted to conventional strike platforms (replacing 24 Trident missiles with ~140 Tomahawk missiles), the payload weight dropped to 206 tons reducing payload fraction to below 3% mainly due to the relative weight difference between a Trident (ballistic) and a

Tomahawk missile. In addition, the ballistic missiles require a very specific hosting interface, which does not easily lend itself to reconfiguration for other payloads.

3. Future Payload Requirements

As autonomous vehicles continue to advance, they will play an increasing role in extending the reach of the U.S. military. Therefore, hosting of UUVs and AUVs onboard submarines becomes a necessity. In order to take full advantage of UUVs and AUVs, autonomous vehicle servicing must be done in theatre. These UUVs and AUVs need to be launched, recovered, repowered, and reconfigured as close as possible to the operation point to maximize their use on station.

For this study, four payloads are evaluated in four potential locations. Payload One is a standard torpedo, Payload Two is a typical missile such as a Tomahawk, Payloads Three and Four are selected from the RAND (Button et al. 2009) study which assesses various UUVs to support UUVMP missions. Bluefin-21, shown in Figure 15, is selected as Payload Three and is representative of medium (heavy weight) UUVs which can be launched from a normal torpedo tube. The Bluefin-21 has a length of 16.2 feet, a diameter of 21 inches, and a displacement of ~1650 lbs (General Dynamics n.d.). SEAHORSE AUV, shown in Figure 16, is selected as Payload Four and is representative of large UUVs/AUVs which are too large in diameter for normal torpedo tubes. The SEAHORSE AUV has a length of 28 feet, a diameter of 36 inches, and a displacement of ~10,500 lbs (AUVAC n.d.).

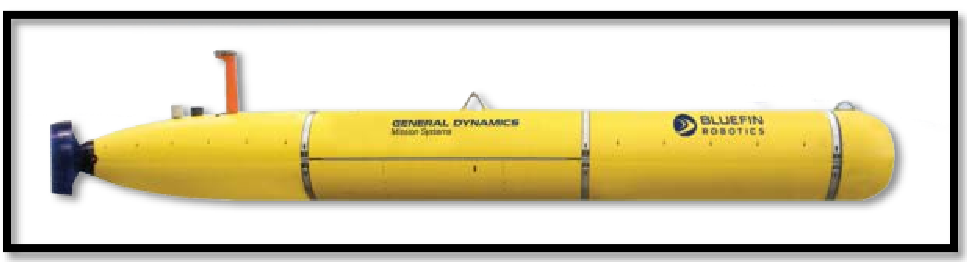


Figure 15. Bluefin-21 Heavy Weight UUV. Source: General Dynamics (n.d.).

Figure 16. SEAHORSE AUV. Source: AUVAC (n.d.).

The four potential hosting locations are taken from existing attack submarine interfaces and the BMT study as follows: 1) Internal payload rooms similar to torpedo rooms, 2) large diameter tubes internal to the pressure hull (as in the Virginia Block 5 VPM), 3) large diameter tubes external to the pressure hull (as in the Virginia Block 3), and 4) wet hangars (Hardy and Barlow 2008). A torpedo room is a space internal to the pressure hull which allows weapons to be loaded into torpedo tubes of 21 inches in diameter which interface with the ocean. For payloads larger than 21 inches in diameter, a larger ocean interface is required. The large diameter tubes are capable of hosting multiple payloads dependent on the payload diameter. Large diameter tubes are designed to keep payloads dry and contain hatches to interface with the ocean. Large diameter tubes internal to the pressure hull have the added benefit of allowing limited user access to the payload for maintenance. Wet hangars are structures external to the pressure hull designed for a specific payload. For this study, each payload housed in a wet hangar requires its own structure. Wet hangars as well as external large diameter tubes do not provide manned access to the payloads. In some cases, it is not prudent or feasible to investigate a specific payload/location combination. For example, torpedoes in a missile tube or missiles internal to the pressure hull without a tube interface. Table 2 shows the payload characteristics and locations for this study. Greyed out lines are not feasible payload/location combinations for this study.

Table 2. Payload Characteristics / Configurations

Payload	Location	Weight (lbs)	Length (ft)	Diameter (ft)	Volume (Ft^3)
Torpedo	Int Room	3434	19	1.75	45.7
	Int Tube	3434	19	1.75	45.7
	Ext Tube	3434	19	1.75	45.7
	Wet Hangar	3434	19	1.75	45.7
Missile (Tomahawk)	Int Room	3300	20.5	1.67	44.9
	Int Tube	3300	20.5	1.67	44.9
	Ext Tube	3300	20.5	1.67	44.9
	Wet Hangar	3300	20.5	1.67	44.9
Bluefin 21 (HW UUV)	Int Room	1650	16.2	1.75	39.0
	Int Tube	1650	16.2	1.75	39.0
	Ext Tube	1650	16.2	1.75	39.0
	Wet Hangar	1650	16.2	1.75	39.0
Seahorse (Large AUV)	Int Room	10500	28	3	197.9
	Int Tube	10500	28	3	197.9
	Ext Tube	10500	28	3	197.9
	Wet Hangar	10500	28	3	197.9

Payload characteristics (length, diameter, and weight) and hosting locations.

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III. METHODOLOGY

A. MODEL SETUP

The goal of this analysis is to develop trends and system parameters for how different types of payloads impact ship design in terms of overall displacement, which is a proxy for cost. The optimization model developed as part of this thesis prescribes the best values for the number of payloads, location of payloads, submarine length, and submarine diameter that maximizes the payload fraction.

B. SUBMARINE DESIGN MODEL

The submarine design model built for this thesis is a weights-based model. It has four distinct modules: the Hull Geometry, the Weight Table, the Payload Table, and the Payload Fraction. Each module is discussed in more detail below.

1. Hull Geometry

The Hull Geometry module calculates the submarine pressure hull volume (V_{PH}) and therefore the available buoyance. This module includes two parameters of length and diameter. The length is broken down into L_a, L_{PMB}, and L_f in accordance with Figure 3 and utilizes the suggested L/D ratio of six. Equations (1.2) and (1.3) are used to calculate the hull offsets (y_f, y_a) utilizing nominal values for n_a and n_f of 3 and 2.75 respectively. The offsets for the PMB are simply calculated as $D/2$. Finally, Equation (1.5) is used to estimate V_{form} and Equation (1.8) to calculate the V_{PH} with a FF of 15% and a MBT of 12.5%.

2. Weight Table

The Weight Table module calculates the weight of each Weight Group, which is summed and compared to the available ship buoyancy. The weight table, shown in Table 3, utilizes the same categories as Table 1. A simplifying assumption made for this thesis assigns all payload weight (including required ballast and structural weight) to WG_7. Typically, the payload weight is captured in the variable load category. For all other weight groups, estimations are made based on ship characteristics (length and diameter). Weight

Group 1 (Hull Structure), Weight Group 4 (Command/Control), Weight Group 5 (Auxiliary), Weight Group 6 (Outfitting), Lead, and Variable Load are set to 39%, 4%, 10%, 4%, 16%, and 4% of *NSC* respectively, to be consistent with the example concept presented by Jackson (1994). Weight Group 2 (Mechanical) and 3 (Electrical) are a function of shaft horse power (*Power*) and calculated using Equations (1.10) and (1.11) from Burcher and Rydill (1994) and Powell (1958). Burcher and Rydill (1994) suggest a power factor (K_p) value of 20 but allow K_p to increase if the length to diameter increases over ideal values. For this study, K_p is increased to 30 to account for the larger L/D ratios. Shaft and drive train inefficiencies captured by η_0 *(power)* x η_H *(motor)* x η_S (shaft) are set to 0.735 based on Burcher and Rydill (1994). U_{max} is user defined and represents the maximum submerged speed which is set to 20 knots for this study. Calculating Weight Group 7 is the end result of the Weight Table module.

$$Power = \frac{K_p \times V_{form}^{0.64} \times U_{\max}^{2.9}}{\eta_0 \times \eta_H \times \eta_S} \tag{1.10}$$

$$WG_2 + WG_3 = 242 \times \left(\frac{Power}{1000}\right)^{\frac{1}{3}} \tag{1.11}$$

Table 3. Weight Table Module

Weight Balance Title	**Symbol**	**Notes**
Weight Group 1	WG_1	0.39% *NSC* (Source: Jackson)
Weight Group 2	WG_2	Calculated using (1.10) Source: Burcher and Rydill and (1.11) Source: Powell
Weight Group 3	WG_3	
Weight Group 4	WG_4	0.04% *NSC* (Source: Jackson)
Weight Group 5	WG_5	0.10% *NSC* (Source: Jackson)
Weight Group 6	WG_6	0.04% *NSC* (Source: Jackson)
Weight Group 7	WG_7	Calculated using (1.13)
Lead	*Ld*	16% of sum $WG_{(1-7)}$ (Source: Jackson)
Variable Load	*VL*	4% of *NSC* (Source: Jackson)
Near Surface Condition	*NSC*	Set equal to V_{PH}/ 35 (ft^3/ton)

Submarine weight categories and the associated assumptions for conducting design optimization.

Equation (1.9) provides a relationship between the Weight Groups and the near surface condition (*NSC*). This equation is solved for WG_7 as shown in Equation (1.12).

$$WG_7 = \frac{NSC \times [1 - \%VL - [\%WG_1 \times [1 + \%Ld]]]}{[1 + \%Ld]} - \sum_{i=2}^{6} WG_i \qquad (1.12)$$

Since Weight Groups 1–6 are represented as a percentage or by actual values, then Equation (1.12) is re-written as Equation (1.13) to be consistent with Table 3. This represents the "Available" value for Weight Group 7.

$$WG_7 = NSC \times \left[\frac{[1 - \%VL - [\%WG_1 \times [1 + \%Ld]]]}{[1 + \%Ld]} - \sum_{i=4}^{6} \%WG_i \right] - \sum_{i=2}^{3} WG_i \qquad (1.13)$$

3. Payload Module

The Payload Module defines the total payload that can be hosted within the available weight margin. The four payloads and hosting locations are defined in Chapter II. The number of payload *i* at location *j* is represented by N_{ij}. The number of payload host structures for payload *i* at location *j* is represented by T_{ij}. The payload's weight is defined as W_{ij}. For each payload, the ship is required to account for ballast weight B_{ij}. Ballast weight is used to keep the ship balanced when a payload leaves the vessel. In some cases, the space occupied by the payload is filled with seawater once the payload has left the host which reduces the amount of ballast required. Each payload case (type / location) results in support structure required, $S1_{ij}$. In other cases, that weight is shared across payloads, (e.g., an internal torpedo room structure would be common) $S2_{ij}$. Due to the limited published data on submarine payload interface, the submarine support structure weight was estimated based on a volume slightly larger than the payload and the weight of steel, while common structure was calculated based on the estimated size required to support multiple payloads and the weight of steel.

Each payload also has an associated unit volume that must be accounted for (defined as $V1_{ij}$) and, in cases where hosting structure is common, a common volume is also required (defined as $V2_{ij}$). Table 4 shows the values for estimating the weight and volume impact for each payload.

Table 4. Payload Table Module

							Weight (lbs)			Volume (ft^3)	
Payload	Location	Weight (lbs)	Length (ft)	Diameter (ft)	Volume (Ft^3)		Ballast	Individual Structure	Common Structure	Internal	External
Torpedo	Int Room	3434	19	1.75	45.7	1	3434	4272	17053	0	0
	Int Tube	3434	19	1.75	45.7	1	509	4272	27462	1259	0
	Ext Tube	3434	19	1.75	45.7	1	509	4272	27462	8.0	1259
	Wet Hangar	3434	19	1.75	45.7	1	509	8544	0	8.0	91
Missile (Tomahawk)	Int Room	3300	20.5	1.67	44.9	1	3500	4398	18400	0	0
	Int Tube	3300	20.5	1.67	44.9	1	426	4398	0	0	0
	Ext Tube	3300	20.5	1.67	44.9	1	426	4398	0	6.7	0
	Wet Hangar	3300	20.5	1.67	44.9	1	426	8797	0	6.7	90
Bluefin 21 (HW UUV)	Int Room	1650	16.2	1.75	39.0	1	12000	3642	29080	0	0
	Int Tube	1650	16.2	1.75	39.0	1	844	3642	0	0	0
	Ext Tube	1650	16.2	1.75	39.0	1	844	3642	0	13.2	0
	Wet Hangar	1650	16.2	1.75	39.0	1	844	7285	0	13.2	78
Seahorse (Large AUV)	Int Room	10500	28	3	197.9	1	30000	10792	100525	0	0
	Int Tube	10500	28	3	197.9	1	2167	10792	0	0	0
	Ext Tube	10500	28	3	197.9	1	2167	10792	0	33.85784	0
	Wet Hangar	10500	28	3	197.9	1	2167	21584	0	33.85784	396

Payload characteristics along with hosting locations and estimated for weight and volumetric impacts to the host platform used in this study.

Payload weight accounts only for the payload itself as shown in Equation (1.14).

$$W_{Payload} = \sum_{i=1}^{4}\sum_{j=1}^{4} N_{ij} \times W_{ij} \tag{1.14}$$

Payload hosted weight accounts for the payload weight plus the ballast (B_{ij}) and the individual and common structure ($S1_{ij}$ and $S2_{ij}$) as shown in Equation (1.15).

$$W_{PL\_Hosted} = \sum_{i=1}^{4}\sum_{j=1}^{4} N_{ij} \times (W_{ij} + B_{ij} + S1_{ij}) + \sum_{i=1}^{4}\sum_{j=1}^{4} T_{ij} \times S2_{ij} \tag{1.15}$$

Internal volume accounts for the volume of payload located internal to the pressure hull (j=1 and j=2) as shown in Equation (1.16). Burcher and Rydill (1994) suggests that the ratio of internal payload volume to pressure hull volume should equal 30% as shown in equation (1.17).

$$V_{PL\_Int} = \sum_{i=1}^{4}\sum_{j=1}^{2} N_{ij} \times V1_{ij} + \sum_{i=1}^{4}\sum_{j=1}^{2} T_{ij} \times V2_{ij} \tag{1.16}$$

$$\mathrm{V}_{\mathrm{PL\_Int}} = 0.3 \times V_{PH} \tag{1.17}$$

External volume accounts for the volume of payloads located outside the pressure hull (*j*=3 and *j*=4) as shown in Equation (1.18).

$$V_{PL\_Ext} = \sum_{i=1}^{4}\sum_{j=3}^{4} T_{ij} \times V2_{ij} \tag{1.18}$$

If payloads are located external to the pressure hull, Equation (1.8) used for calculating $V_{PH,}$ requires updating to account for the fact that external payloads displace main ballast tank volume needed for reserve buoyance. To account for required volume to make up for external payloads, V_{PL_Ext} is subtracted from V_{PH} as shown in Equation (1.19).

$$V_{PH} = \frac{V_{form}}{1.15 \times (1 + ROB / 0.98)} - \sum_{i=1}^{4}\sum_{j=3}^{4} T_{ij} \times V2_{ij} \tag{1.19}$$

4. Payload Fraction

The Payload Fraction module is the final check and compares the sum of the payload weight (including all ballast and structure required) with the “Available” Weight Group 7 value from Equation (1.13). If “Available” WG7 is larger than the total payload weight, then the design can be balanced. Then the payload fraction is calculated as the ratio of the sum of payload weight to the *NSC*.

C. MODEL ASSUMPTIONS AND LIMITATIONS

This model is a first-order tool to aid in submarine concept design; therefore, there are several factors not accounted for in this model.

First, the model is focusing on balancing the submarine statically to be neutrally buoyant in a single condition. As submarines operate during a mission, they consume material, expend weapons and fuel, ingest and expel seawater. This requires multiple loading conditions and is typically addressed with trim and ballasting tanks.

Second, an assumption is made that the longitudinal center of gravity and the longitudinal center of buoyancy match. In a final submarine design, lead ballast is often required to be placed along the submarine keel to ensure the ship is not only neutrally buoyant but also longitudinally balanced.

Third, the model does not account for submarine deck layouts (equipment, berthing, etc.) which can drive the design to be less efficient in terms of payload fraction.

Finally, the model uses open source data in order to avoid classification. While the design methods referenced are sound, the factors and values may not fully reflect the state of the art in submarine design.

D. MODEL FORMULATION

1. Indices

i — i denotes the payload type (1 = Torpedo, 2 = Tomahawk, 3 = Medium UUV/AUV, 4 = Large UUV/AUV)

j — Payload location where j denotes the hosted location on the submarine (1 = Internal Room, 2 = Internal Tube, 3 = External Tube, 4 = Wet Hangar)

2. Parameters and Data [Units]

W_{ij} — Weight of Payload i at Location j [Long Tons]

B_{ij} — Ballast Weight required for each Payload i at Location j [Long Tons]

$S1_{ij}$ — Structural Weight required for each Payload i at Location j [Long Tons]

$S2_{ij}$ — Common Structural Weight required for Payload i at Location j [Long Tons]

$V1_{ij}$ — Volume required for each Payload i at Location j [Cubic Feet]

$V2_{ij}$ — Common Volume required for Payload i at Location j [Cubic Feet]

M_{ij} — Maximum Number of Payload i that can be hosted in a common structure

M'_{ij} — Maximum Number of Payload i that can be at location j (set to 999)

V_{PH} — Volume of the Pressure Hull (see Equation (1.19)) [Cubic Feet]

WG_7 — Available weight for payloads (see Equation (1.13)) [Long Tons]

ROB — Reserve Buoyancy Fraction

n_f Geometric parameter that determines the fullness of the bow

n_a Geometric parameter that determines the fullness of the stern

$Nmax_{ij}$ Maximum Number of Payload *i* at Location *j* allowed (user defined)

$Nmin_{ij}$ Minimum Number of Payload *i* at Location *j* required (user defined)

3. Decision Variables

The decision variables are partitioned into two types. The first type are the variables length (*L*) and diameter (*D*), associated with the ship geometry. These dimensions are the critical parameters for calculating the envelope displacement. Length is made up of the forward body, the aft body, and the parallel mid body. The ship displacement is calculated by integrating the hull lines. The envelope displacement is then used to calculate *NSC* by subtracting for free-flood area and for main ballast tanks. The second type of variable is associated with the payloads and includes two variables. N_{ij} is the number of payloads and T_{ij} is the number of payload host structures. Both variables are integers. $T1_{ij}$ is a binary variable used to identify if payloads are present at each location. The total number of payloads N_{ij} must not exceed the capacity of the total number of host structures T_{ij}.

In summary, the decision variables are:

N_{ij} Number of Payload *i* at Location *j*

T_{ij} Number of Payload Host Structures for Payload *i* at Location *j*

$T1_{ij}$ Binary variable where 1 denotes the presence of Payload *i* at Location *j, and 0 otherwise*

L Submarine Length

D Submarine Diameter

4. Integer Non-linear Programming Model

The overall objective is to maximize payload fraction by varying payload numbers (N_{ij}), payload hosting structures (T_{ij}), payload presence ($T1_{ij}$), and ship parameters length

(L) and diameter (D). Integer Non-linear Program 1 (*INLP1*) maximizes the payload fraction in Equation 1.20.

$$MAX\left[\frac{\sum_{i=1}^{4}\sum_{j=1}^{4} N_{ij}\times W_{ij}}{\left(\frac{\sum_{x_f=0}^{3.6D}\pi\left(\frac{D}{2}\left[1-\left(\frac{x_f}{3.6D}\right)^{n_f}\right]^{\frac{1}{n_f}}\right)^2+\sum_{0}^{L-6D}\pi\left(\frac{D}{2}\right)^2+\sum_{x_a=0}^{2.4D}\pi\left(\frac{D}{2}\left[1-\left(\frac{x_a}{2.4D}\right)^{n_a}\right]\right)^2}{1.15\times\left(1+\frac{ROB}{0.98}\right)}-\sum_{i=1}^{4}\sum_{j=3}^{4}T_{ij}\times V2_{ij}\right)/35}\right] \tag{1.20}$$

subject to:

$$W_{PL\_Hosted}\le WG_7\ . \tag{1.21}$$

Constraint (1.21) insures that the total weight of the payloads and their supporting structure W_{PL_Hosted} calculated from Equation (1.15) does not exceed the “Available” weight allocated for WG_7 calculated from Equation (1.13).

$$V_{PL\_Int}\le 0.3\times V_{PH} \tag{1.22}$$

Constraint (1.22) ensure that the total internal pressure hull volume required by the payloads and their supporting structures calculated from Equation (1.16) does not exceed 30 percent of the available pressure hull volume as suggested by Burcher and Rydill (1994) in Equation (1.17).

$$N_{ij}\le M_{ij}\times T_{ij}\quad \forall i,j \tag{1.23}$$

Constraint (1.23) ensures that the number of payloads, N_{ij}, does not exceed the capacity of the payload support structure, M_{ij}, and also ensures that a sufficient number of payload support structures, T_{ij}, exist.

$$N_{ij} \leq M'_{ij} \times T1_{ij} \quad \forall i, j \tag{1.24}$$

$$\sum_{j=1}^{4} T_{ij} \leq 1 \quad \forall i \tag{1.25}$$

Constraint (1.24) limits the number of payloads allowed in each location to no more than M'_{ij} if a payload of type *i* exists in location *j*. Constraint (1.25) ensures that the number of payload support structures T_{ij} can only be sited at one location *j* for each payload type *i*.

$$L \leq 15 \times D \tag{1.26}$$

$$L \geq 6 \times D \tag{1.27}$$

Constraints (1.26) and (1.27) ensure that the ratio of *L* to *D* is within good submarine design practice as discussed in Chapter II. A *L*/*D* ratio of six is optimal and current designs do not exceed 15.

$$N_{ij} \leq N\max_{ij} \quad \forall i, j \tag{1.28}$$

$$N_{ij} \geq N\min_{ij} \quad \forall i, j \tag{1.29}$$

Constraints (1.28) and (1.29) are user-defined constraints on the number of each payload N_{ij} based on user parameter $Nmax_{ij}$ and $Nmin_{ij}$.

$$Nij \geq 0 \quad \forall i, j \tag{1.30}$$

$$N_{ij} = Integer \quad \forall i, j \tag{1.31}$$

$$T_{ij} = Integer \quad \forall i, j \tag{1.32}$$

$$T1_{ij} = Binary \quad \forall i, j \tag{1.33}$$

Constraints (1.30), (1.31), (1.32), and (1.33) provide integer, binary, and non-zero restrictions.

The objective function is a non-linear integer equation due to the N_{ij} variable being divided by D, therefore, a simplification is made to reduce computational complexity. Upper and lower bounds on the diameter, D, are set based on typical designs, while the length, L, is bound by Equations (1.26) and (1.27). For diameter, 32 feet is set as a lower bound and considered the minimal required to allow for adequate deck space inside the pressure hull while 44 feet is set as an upper bound which is 2 feet larger than current U.S. Navy submarines. For length, 320 feet is set as a lower bound which is 40 feet shorter than current U.S. Navy submarines while 660 feet is set as on upper bound based on a maximum diameter of 44 feet and the L/D relationship in Equation (1.26).

By setting L and D as user defined parameters, the objective is no longer a non-linear function of the decision variables and is reduced to maximizing payload weight as shown in Equation (1.34). Also, since L and D are now determined parametrically, any constraints on L and D in the formulation are removed and those restrictions are accounted for in the determination of the appropriated L and D parameters.

Simplifying the objective function and dropping constraints (1.26) and (1.27) results in Integer Linear Program 2 (*ILP2*) defined in Equation (1.34). Discussion of the implementation follows in the next chapter.

$$Maximize \quad \sum_{i=1}^{4}\sum_{j=1}^{4} N_{ij} \times W_{ij} \tag{1.34}$$

IV. ANALYSIS

A. INTRODUCTION

This chapter explains how the optimization model, *ILP2*, is implemented, reviews the model validation, discusses the use of optimization to assess payload trends, and shows how the model is used to maximize payload fraction for a series of specific payload conditions.

B. MODEL IMPLEMENTATION

The model *ILP2* formulation from Chapter III is implemented in Microsoft Excel using the Solver add-in. The data entry table created in the Payload Module allows the user to define parameters for minimum payload limit, $Nmin_{ij}$, and the maximum payload limit, $Nmax_{ij}$, which are shaded light blue in Table 5. The decision variables N_{ij}, T_{ij}, and $T1_{ij}$ are the unknown values determined by the Excel solver add-in.

To begin a model run, the user defines the limits on payloads. For example, most submarines require a minimum number of torpedoes for self-defense. To accomplish this, $Nmin_{11}$ (i=1:Torpedo, j=1:Internal Room) and $Nmin_{14}$ (i=1:Torpedo, j=4:Wet Hangar) are set to the desired minimum as shown in Table 5. $Nmax_{11}$ and $Nmax_{14}$ are set to 999 to provide an upper limit to the solver. This allows the model to satisfy the minimum torpedo requirement using either an Internal Room or Wet Hangars. For payload / location combinations that are not desired, $Nmin_{ij}$ and $Nmax_{ij}$ are set to zero. For example, if torpedoes in the wet hangar location are not desired, $Nmin_{14}$ (i=1:Torpedo, j=4:Wet Hangar) and $Nmax_{14}$ (i=1:Torpedo, j=1:Wet Hangar) would be set to zero.

Table 5. Payload Constraint Module

Payload				Payload Variable			Characteristics				User Constraints	
Name	i	Location	j	N_{ij}	T_{ij}	$T1_{ij}$	Length (ft)	Diameter (ft)	Volume (ft^3)		$Nmin_{ij}$	$Nmax_{ij}$
Torpedo	1	Int Room	1				19	1.75	45.7		24	999
		Int Tube	2	Not Considered								
		Ext Tube	3	Not Considered								
		Wet Hangar	4				19	1.75	45.7		24	999
Missile (Tomahawk)	2	Int Room	1	Not Considered								
		Int Tube	2				20.5	1.67	44.9		0	0
		Ext Tube	3				20.5	1.67	44.9		0	0
		Wet Hangar	4				20.5	1.67	44.9		0	0
Bluefin 21 (HW UUV)	3	Int Room	1				16.2	1.75	39.0		0	0
		Int Tube	2				16.2	1.75	39.0		0	0
		Ext Tube	3				16.2	1.75	39.0		0	0
		Wet Hangar	4				16.2	1.75	39.0		0	0
Seahorse (Large AUV)	4	Int Room	1				28	3	197.9		0	0
		Int Tube	2				28	3	197.9		0	0
		Ext Tube	3				28	3	197.9		0	0
		Wet Hangar	4				28	3	197.9		0	0

The Payload Module Table allows user defined parameters for minimum and maximum payload limits ($Nmin_{ij}$, $Nmax_{ij}$) which are shaded light blue. In this case, a minimum of 24 torpedoes is desired.

The length and diameter parameters are pre-determined and entered into the Hull Geometry model, which is used to calculate the envelope displacement. The diameter, D, is bound between 32 and 44 feet based on the arrangement and infrastructure restrictions. D is varied by two-foot increments resulting in seven potential values.

The length, L, is bound by Equations (1.26) and (1.27), establishing a range in length of 198 ft (6 x 32) to 660 ft (15 x 44). The shortest U.S. fleet submarine is the Los Angeles class at 360 feet. Lengths below 320 feet are ignored based on current practice because there is insufficient volume available for payloads. Dividing L into 20 foot increments results in between 9 and 18 integer increments per D. The resulting parameter lattice results in 93 feasible combinations of L and D.

After the user parameters are set in the Payload Constraint and Hull Geometry modules, an Excel macro executes the Excel Solver add-in for all feasible L and D combinations and populates the Payload Fraction module with the output data, L, D, N_{ij},

W_{PL_Hosted}, *NSC*, and *Payload Fraction*. This tabular data is available to the user for additional analysis.

For each payload configuration, the model is executed 93 times through programming macros in Excel. A Visual Basic macro was developed to run the optimization model on a loop through all feasible *L* and *D* combinations. Total run time for a single *L* and *D* combination for a given set of user constraints is less than one minute on average. Certain configurations of user constraints result in longer run time of up to five minutes and when using the Excel macro to run through the *L* and *D* combinations, the average total run time for a payload configuration is 45 minutes to 1 hour.

To reduce run time, more stressing cases were run multiple times with different integer optimality gaps. The optimality gap as defined by Mathematical Programming Glossary is "the difference between the best known solution and a value that bounds the best possible solution" (Mathematical 2017). Run time for a stressing *L* and *D* combination with an integer optimality of less than 5% is on the order of five minutes. Increasing the optimality gap to between 5% and 10% reduces run times to less than three minutes without changing the resulting solution. Increasing optimality beyond 10% to 12% reduces run times to less than one minute but the resulting solutions returned by the Excel Solver are known to be suboptimal because the resulting objective function values are larger than those found with the tighter optimally gap. As a result, the optimality for the model is set at 5% as the ~2-minute time savings per run adds up to two to three hours in savings per payload loadout.

C. VALIDATION CASE

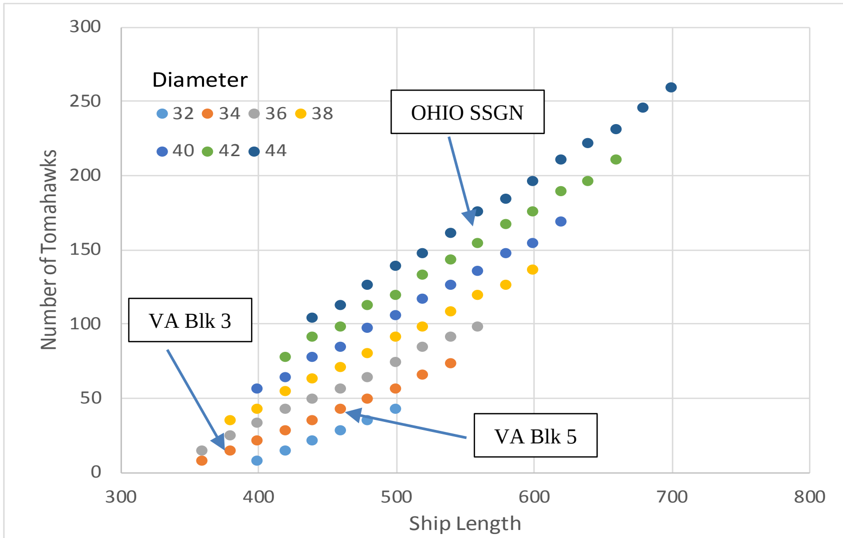
Three test cases are selected to validate the optimization model's accuracy against existing submarine designs. The test cases are the Virginia class Block 3, the Virginia class Block 5, and the Ohio class SSGN conversion. Since submarine designs to date are focused on weapons (torpedoes and missiles), there are no validation cases for the UUV or AUV payloads. The model is set up with a hard user constraint on the number of torpedoes (set to 24 to match the Virginia and Ohio load outs) represented as $24 <= N_{11} <= 24$. As shown in Table 6, the only other payloads allowed to vary are N_{22} and N_{23} represented the tube hosted Tomahawks.

Table 6. Validation Case User Constraints

Payload					Characteristics				User Constraints	
Name	i	Location	j		Length (ft)	Diameter (ft)	Volume (ft^3)		Nij Min	Nij Max
Torpedo	1	Int Room	1		19	1.75	45.7		24	24
		Int Tube	2		Not Considered					
		Ext Tube	3		Not Considered					
		Wet Hangar	4		19	1.75	45.7		0	0
Missile (Tomahawk)	2	Int Room	1		Not Considered					
		Int Tube	2		20.5	1.67	44.9		0	999
		Ext Tube	3		20.5	1.67	44.9		0	999
		Wet Hangar	4		20.5	1.67	44.9		0	0
Bluefin 21 (HW UUV)	3	Int Room	1		16.2	1.75	39.0		0	0
		Int Tube	2		16.2	1.75	39.0		0	0
		Ext Tube	3		16.2	1.75	39.0		0	0
		Wet Hangar	4		16.2	1.75	39.0		0	0
Seahorse (Large AUV)	4	Int Room	1		28	3	197.9		0	0
		Int Tube	2		28	3	197.9		0	0
		Ext Tube	3		28	3	197.9		0	0
		Wet Hangar	4		28	3	197.9		0	0

The Payload Module Table for the validation run with parameters set for 24 torpedoes and the ability to optimize the remaining payload volume for missile in internal or external large diameter tubes.

Using the data listed in Table 6, the optimization model runs over a range of diameters between 32 and 44 feet and a range of lengths between 320 and 660 feet. Figure 17 shows that the model optimizes designs that are consistent with VIRIGNIA and Ohio test cases. The Virginia Block 3 is 377 feet long and 34 feet in diameter and has a Tomahawk loadout of 12. For a 380-foot length at 34-foot diameter, the model returns a value of 14 Tomahawks. The Virginia Block 5 is 461 feet long and 34 feet in diameter and has a Tomahawk loadout of 40. For a 460-foot length at 34-foot diameter, the model returns a value of 42 Tomahawks. The delta of two Tomahawks can be accounted for with a design constraint of the Virginia that limits the first two missile tubes to 6 Tomahawks versus 7. The Ohio SSGN conversion is 560 feet long and 42 feet in diameter and has a Tomahawk loadout of 154. For a 560-foot length at 42-foot diameter, the model returns a value of exactly 154 Tomahawks.



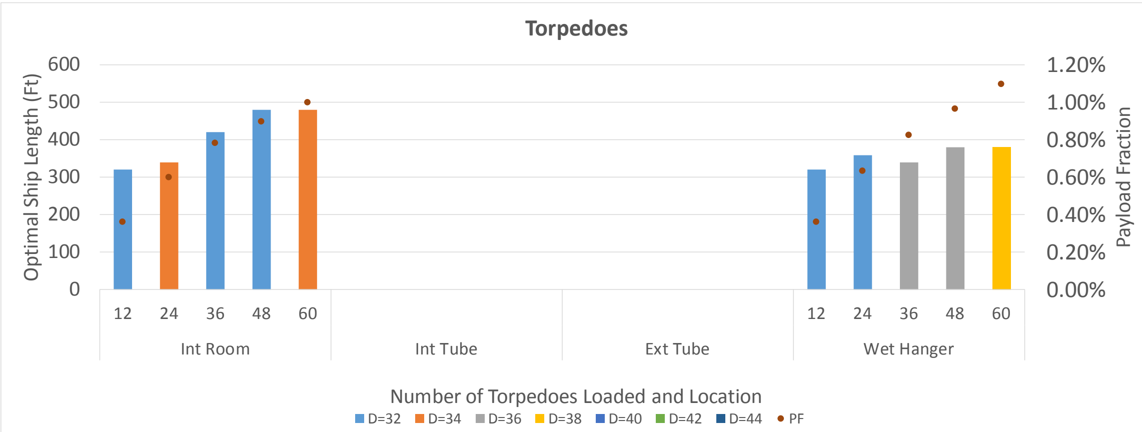
Optimized missile load outs with 24 torpedoes run as a validation case against the payload optimization model. For a given *L* and *D*, the model returns the maximum number of missiles that can be hosted. The data points are colored to show the associated diameter. VA Blk 3 (Virginia Block 3) is 377 feet with a 34-foot diameter, VA Blk 5 (Virginia Block 5) is 460 feet with a 34-foot diameter, and Ohio SSGN is 560 feet with a 42-foot diameter.

Figure 17. Tomahawk Trends (Tube Hosted, with 24 Torpedoes)

D. PAYLOAD TREND ANALYSIS

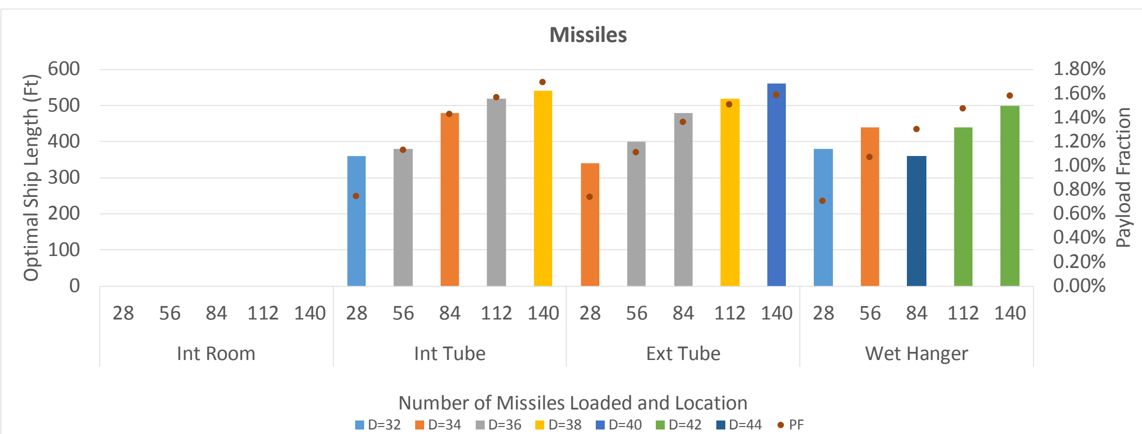
The optimization model is also used to assess trends in payload fraction for a given payload. This data is used as a decision aide during concept design to help designers understand the impacts of location for payload as well as any changes in the trend of payload capacity (for a given payload type and location) with respect to length and diameter.

By setting all the user defined parameter for maximum payload ($Nmax_{ij}$) to zero for all but one payload, a trend is developed for payload fraction by increasing diameter and length. For example, incrementing $Nmax_{11}$ from 12 by increments of 12 and rerunning the model, while setting all other $Nmax_{ij}$ to zero, the model determines the optimal length and diameter to host each torpedo load out case. Figures 18 through 21 show how the optimal length and diameter change as each payload is individually incremented for each viable hosted location. Length bars are plotted against the left hand y-axis and the color of the bar is associated with the submarine diameter. The payload fraction, represented by the dots on the graph for each bar, is plotted against the right hand y-axis.



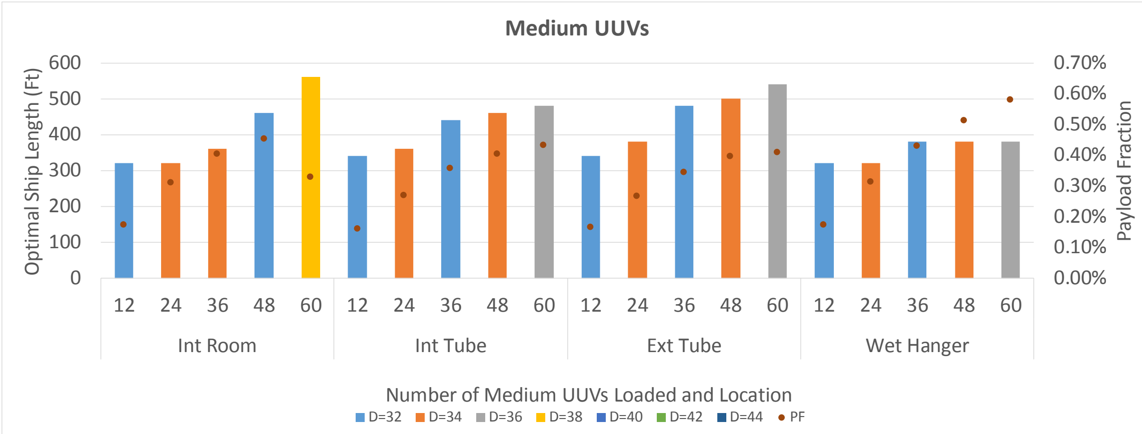
Length and diameter to maximize payload fraction for a desired amount of torpedoes at a given location (Internal and External Tubes are not used). As the number of payloads increase, length bars are plotted against the left hand y-axis and the bar color changes with diameter. The black data point shows the associated payload fraction are plotted against the right hand y-axis.

Figure 18. Torpedo Loadout Trends



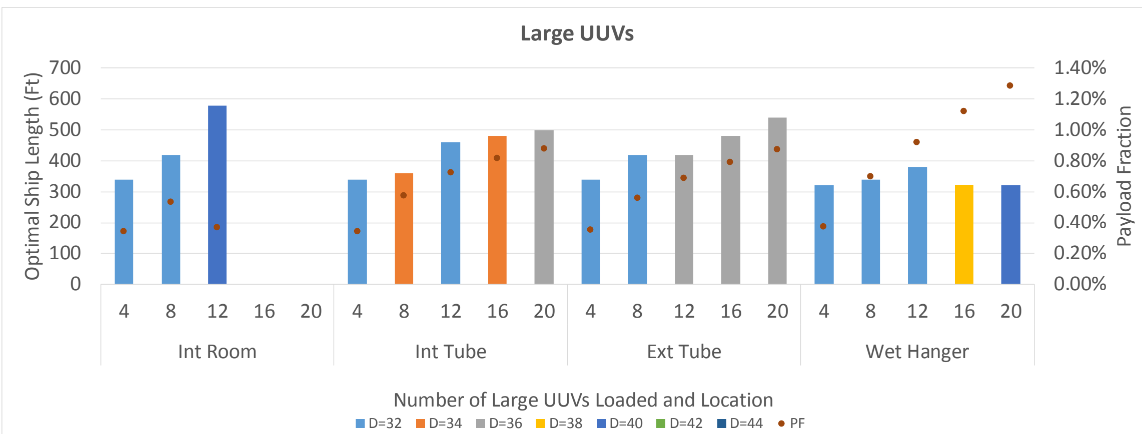
Length and diameter to maximize payload fraction for a desired amount of missiles at a given location (Internal Rooms are not used). As the number of payloads increase, length bars are plotted against the left hand y-axis and the bar color changes with diameter. The black data point shows the associated payload fraction are plotted against the right hand y-axis.

Figure 19. Missile Loadout Trends



Length and diameter to maximize payload fraction for a desired amount of medium UUVs at a given location. As the number of payloads increase, length bars are plotted against the left hand y-axis and the bar color changes with diameter. The black data point shows the associated payload fraction are plotted against the right hand y-axis.

Figure 20. Medium UUV Loadout Trends



Length and diameter to maximize payload fraction for a desired amount of large UUVs at a given location. As the number of payloads increase, length bars are plotted against the left hand y-axis and the bar color changes with diameter. The black data point shows the associated payload fraction are plotted against the right hand y-axis.

Figure 21. Large UUV Loadout Trends

The following observations are made from the payload trends analysis with data generated using the optimization model.

1. Payload Fraction versus Payload Loadouts

As the number of payloads hosted increase, the ship grows to accommodate the additional payload. However, the volumetric efficiency increases as well. This results in the payload fraction increasing with payload loadout. This follows as a minimum amount of the volume of the submarine is dedicated to weight groups 1 through 6 and as the ship grows beyond what is required for these weight groups, more space is available for payloads. The results also show that there is a point of diminishing returns as in general the overall increase in payload fraction is reduced as the total payload loadout increases. Extrapolation of the payload fraction data in Figures 18 through 21 shows that the payload fraction appears to reach an asymptote of between 1% to 2%. This matches well with historical designs as discussed in Chapter II. The exception to this result is the use of wet hangars for hosting Large UUVs. Figure 21 show that for the wet hangar location, the slope of the payload fraction versus loadout does not decrease as the number of payloads increase. Therefore, this analysis suggests a payload fraction beyond 2% is plausible for Large UUV loadouts.

2. Length and Diameter Selections

When optimizing payload fraction, the associated values for *D* are driven toward the lower band of the diameter range. Figures 18 through 21 show a total of 63 feasible payload conditions. Of those 63, 39 have values for diameters of 32 or 34 feet. Diameters that are more than 40 feet are associated with an optimal payload fraction only three times (in Figure 19 for missile loadouts of 84, 112, and 140 in wet hangar locations).

For the medium UUV payload loadouts (Figure 20) shows high correlation between length and diameter associated with the optimal payload fraction. For the internal tube, external tube and wet hangar locations, as the number of UUVs increase from 12 to 48, the diameter fluctuates between 32 and 34 feet. When moving from 12 to 24 UUVs, the length change is minor but the diameter increases by two feet to account for the added weight. From 24 to 26 UUVs, the length increase is more drastic as the diameter is decreased back to 32 feet. The tendency for the model to return smaller diameters suggests that minimizing diameter has a larger impact on maximizing payload fraction than minimizing length. A closer look at the impact of diameter and length on *NSC* shows that the *NSC* and therefore payload fraction is more sensitive to diameter. For a 34-foot diameter, 440 foot long submarine, a two-foot increase in diameter provides the same increase in *NSC* (the denominator in the payload fraction calculation) as a 40-foot increase in length.

3. Large UUV and Volume

Figure 21 shows that for internal rooms there are no feasible solutions for large UUV loadouts equal to or greater than 16 payloads. The required volume for hosting 16 large UUVs exceeds 30% of the pressure hull volume which violates constraint (1.22). Hosting payload in internal rooms (inside the pressure hull) can have a disproportional impact on volume. The internal room location shown in Figure 21 reveals that the payload factor peaks with eight large UUVs and then decreases with 12 UUVs. Figure 19 shows a similar result, which suggests that for the medium UUV, the model may be close to reaching a point where the internal payload volume exceeds 30% of the pressure hull volume.

E. OPTIMIZATION OF SPECIFIC LOAD CASES

While the trend analysis is useful for understanding the impacts of a singular payload type on the submarine, it does not represent realistic loadouts. All submarines carry a minimum number of torpedoes for self-defense and the remaining payload capacity is dictated by mission requirements. When submarines are designed the initial payload configuration is normally set in mission requirements. In traditional design cycles, specific payloads are determined by trading off payload capacity with other requirements as the design matures. This section is focused on utilizing the optimization model for determining specific payload loadouts given competing requirements that are not explicitly stated within the optimization model's constraints.

Based on the data from the trend analysis and assessments of where payload loadouts exceed available ship volumes within the defined length and diameter range, a series of 46 representative load conditions are run with the parameters shown in Table 7. The model returned feasible length and diameter combinations for all 46 loadouts, which suggests that the designs are weight limited which means that the pressure hull volume is driven by the need for buoyance to support the submarine's weight. If the designs were volume limited, the model would return less than the desired payload loadouts as the internal payload volume reaches 30% of the pressure hull volume.

Table 7. List of Loadout Conditions

Paylaod	Variable	Values
Torpedo	$Nmax_{1j}$ for j =1,4	12, 24
Missiles	$Nmax_{2j}$ for j =2,3,4	0, 14, 28
Medium UUV	$Nmax_{3j}$ for all j	0, 6, 12, 18
Large UUV	$Nmax_{4j}$ for all j	0, 2

Parameters used to examine the Payload loadout conditions.

The loadout combinations in Table 7 along with the 93 runs per condition results in 4,278 runs which requires over six hours of run time. A summary of the model output data for a selection of configurations is shown in Table 8. The N_{ij} model output is color coded based on location of the payload.

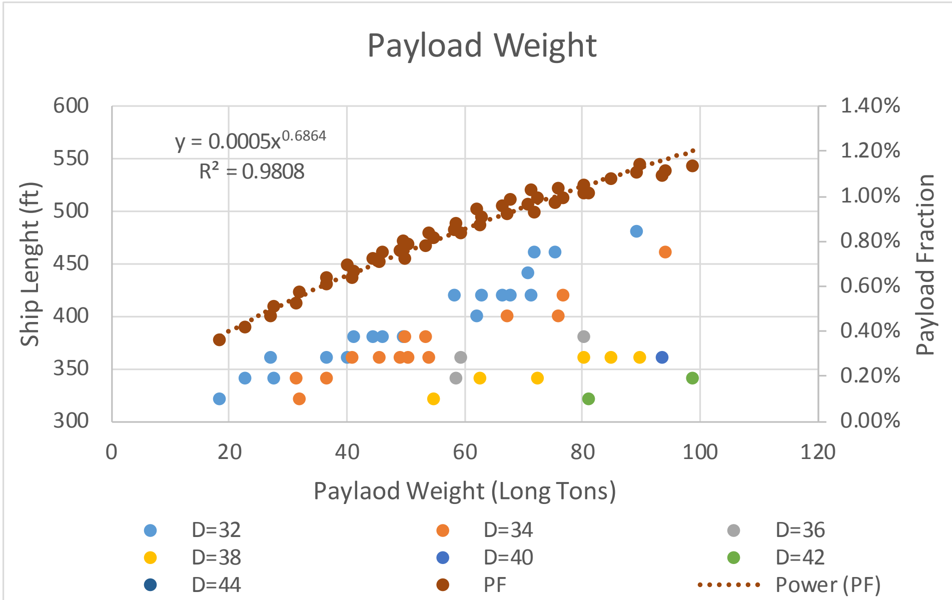
Table 8. Model Outputs for Selected Loadouts

Torpedo	Missile	Med UUV	Large UUV	Length	Diameter	W_{PL_Hosted}	*NSC*	*Payload*
N_{1j}	N_{2j}	N_{3j}	N_{4j}	(Feet)	(Feet)	(Long Tons	(Long Tons	*Fraction*
12	14	6	2	360	34	54.1	6479.6	0.83%
12	14	12	2	420	32	58.5	6899.0	0.85%
12	14	18	2	340	38	62.9	7243.5	0.87%
24	14	6	2	340	38	72.5	7334.8	0.99%
24	14	12	2	420	34	76.9	7776.2	0.99%
24	14	18	2	320	42	81.3	8046.0	1.01%
12	28	12	2	360	38	80.4	7926.9	1.01%
24	28	6	2	460	34	94.3	8490.2	1.11%
24	28	12	2	340	42	98.8	8728.1	1.13%
		Key						
			Internal Room			External Tube		
			Internal Tube			Wet Hanger		

Model outputs for selected loadout conditions. The Length and Diameter represent the most efficient parameters for maximizing payload fraction with the required loadouts. The color indicates the preferred location of the payload.

The selected loadout conditions in Table 8 are some of the more stressing cases in terms of total payload weight. For each configuration, all four payloads are required. In general, wet hangars are preferred locations for torpedoes and UUVs: 9 of 9 for N_{1j}, 6 of 9 for N_{3j}, and 8 of 9 for N_{4j}. The preferred locations for missiles are tubes: 6 of 9 internal and 3 of 9 external. This follows the results of the trends analysis in Section D of this chapter. Figures 18 (torpedoes), 20 (medium UUVs), and 21 (large UUVs) show the highest payload fraction being associated with external hangars. Figure 19 (Missiles) show the highest payload fraction being associated with the tubes. The propensity for the model to utilize wet hangars is inherently biased because payloads exterior to the pressure hull do not grow the pressure hull as much as payloads interior to the pressure hull. Because payload fraction is W_{PL_Hosted} / NSC, and NSC is a function of pressure hull volume, exterior payloads have a greater impact on payload fraction.

Figure 22 shows a summary plot of all the loadout conditions run in terms of payload weight vs payload fraction. In addition, Figure 22 shows the optimized length and diameter for the loadouts.



Optimized submarine conditions for defined loadout conditions. (Left Axis) Optimal Length and Diameter (D is depicted by color) for a given payload weight and (Right Axis) Optimal Payload Fraction for a given payload weight. Payload weight (W_{PL_Hosted}) is the sum of the weight of the desired payload loudout. The associated submarine length and diameter are the optimized value to maximize payload fraction.

Figure 22. Payload Weight vs. Payload Fraction

Consistent with the payload trend analysis, the payload fraction increases as payload weight increases. Additionally, the trend suggests an eventual maximum payload fraction value between 1% and 2%. An initial assumption of this thesis was that optimization methods could increase the payload fraction of submarine designs. Arriving at a 2% ceiling for most payload configurations is only a small increase over the current Virginia class. Some of the rationale for only a minor increase is the fact that the payloads are relatively light and do not dramatically increase the numerator of the payload fraction calculation.

Figure 22 also shows the propensity of the model to return small diameters associated with optimizing payload fraction. Thirty-four of 46 payload conditions have diameter values of 32 to 34 feet while only three had diameters larger than 40 feet. This again shows the large impact of diameter increase on payload fraction.

The loadout conditions are also used to assess which payload hosting locations provide optimal solutions. Table 9 shows how often each payload location is used for each payload type over the 46 loudouts examined. The preferred location for the torpedoes is the wet hangar, which is utilized 89% of the time. For the missiles, it is the internal payload tube utilized 90% of the time. For the medium UUV, the wet hangar is utilized 76% of the time and for the Large UUV, the wet hangar is utilized 95% of the time.

Table 9. Payload Location Usage

Payloads	Int Room	Int Tube	Ext Tube	Wet Hanger
Torpedo	11%	0%	0%	89%
Missile	0%	90%	10%	0%
Med UUV	24%	0%	0%	76%
Large UUV	0%	5%	0%	95%

For the 46 loadout conditions, the percentage of use for each payload location is shown. For example, the wet hangars are used 89% of the time. Note that the internal and external tubes for the torpedoes and the internal room for the missiles are not feasible locations.

The high usage rate of the wet hangar is expected based on the payload trend data from Section D of this chapter. Figures 18, 20, and 21 show that the payload factor for torpedoes, medium UUVs, and large UUVs have the highest potential with the wet hangars. The missile payload is the one exception, which has the highest potential payload factor with internal tube as shown in Figure 19. The payload variables for torpedoes in tubes and missiles in internal rooms are set to zero for these analyses.

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V. CONCLUSIONS

The purpose of this thesis is to investigate the methods to evaluate trade space in payload capacity early in the submarine design phase. Through the use of optimization, the impacts of payload capacity on basic submarine characteristics of length and diameter are assessed. The formulation presented in this thesis is built on sound submarine engineering principles examined extensively by CAPT Harry Jackson and Burchner and Rydill. The model fidelity is hindered slightly by the constraint to use only open source data. However, the real value is in developing a process for using optimization in what has historically been a step-by-step iterative design process.

The formulation in this thesis is modeled in Excel Solver and run as an Integer Linear Program. The model is able to execute thousands of runs within a six-hour period to produce data which can be assessed for trends, narrowing design trade space. Based on the input parameter for the selected payloads (torpedoes, missile, medium UUVs, and large UUVs), the model returns optimal payload fractions between 1 to 2%, which is consistent with historical averages. While this did not produce the anticipated results of large increases to payload fraction, the ceiling of 2% is slightly higher than seen in the Virginia class today. Where validation data was available for torpedo and missile payloads, the model returns optimal length and diameter values consistent with existing fleet designs.

Over 5,000 configurations of payload loadouts, ship lengths, and diameters are assessed in this thesis. The trends analysis in Chapter IV shows that as payload loadouts increase, the ship grows to accommodate and subsequently the payload fraction increases. However, as payload loudouts increase, the trends show diminishing returns as payload fractions approached 2%.

The trends analysis in Chapter IV shows that overall there is a preference for locating payloads in wet hangars outside the pressure hull while maximizing payload fraction. This analysis shows that the added ship length and diameter for increasing payload loadouts is less if the payloads are in wet hangars. This can be expected as payloads exterior to the pressure hull do not increase NSC as much as internal payload do.

Overall the submarine configurations (length and diameter) returned by the model have sufficient volume to host the required numbers of payloads. This suggests a weight limited design meaning that the pressure hull volume is driven by the need for buoyance to support the submarine's weight. Rydill suggests a limit of 30% of pressure hull volume for payloads and this is only a factor in payload loadouts with large UUVs as shown in Figure 21. Future improvements to this model to address arrangements show that volume constrained designs occur more frequently.

All of the 46 loadout conditions' solutions, run to assess more realistic submarine payload loadouts, are feasible. As identified in the trend analysis, Tables 8 and 9 show that wet hangars are the preferred location for payload loadouts. This result may be weakened because the lack of access to a payload located external to the pressure hull is not accounted for in the optimization model. If a payload is to be reliably launched from a host submarine is must be able to withstand its environment for the time of transit. Internal payload structures have the advantage of keeping the payload dry and at a consistent environment (temperature and humidity) with the added benefit of manned access for maintenance and grooming. The data from the 46 loadout cases may inform concept design to place the more reliable payloads in locations with the least access. For example, if all torpedoes are the same and there are 24 on the submarine, locating them exterior to the pressure hull may be acceptable. If there are two large UUVs that have reliability issues, it may be prudent to locate those internal to allow manned access. These decisions can be aided using the optimization model discussed in this thesis.

Future work in developing this optimization model should include: expanding the scope of the optimization to include additional volumetric and arrangement constraints to address some of the model limitations addressed in Chapter III; improving the fidelity of the payload support structure and payload handling system designs; and increasing the suite of available payload and potentially including hosting piggyback vessels such as the ASDS. No matter the improvement made to the model, integration of this form of optimization and potentially the formulation presented in this thesis into U.S. Navy and Department of Defense Contractor proprietary submarine design codes has the potential to more clearly focus the concept design phase of future submarine platforms.

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